RELATIVISTIC EFFECTS IN REFERENCE FRAMES

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ABSTRACT

The impact of relativistic theories of space, time and gravitation on the problem of reference systems is reviewed.

First, the concept of inertial systems is discussed from the point of view of the special and the general theory of relativity. Then, relativistic corrections of Doppler, laser and VLBI, and similar effects are reviewed; they are usually on the order of 10^{-8}. Finally, the problem of a possible variation of the gravitational constant G (on the order of 10^{-11}/year) is outlined; such a variation does not occur in special and general relativity, but is implied by certain generalized field theories which are less commonly accepted.

1. INTRODUCTION

We all know that the special theory of relativity is a refinement of classical mechanics for the case that we are dealing with very high velocities, and that the general theory of relativity provides a refinement of the Newtonian theory of gravitation, relevant for very strong gravitational fields such as the fields of black holes, and for problems of cosmology. For the gravitational field of the Earth and for satellite motion in this field, as well as for terrestrial reference systems, classical mechanics is sufficient; relativistic effects are negligible or can be taken into account by very small corrections.

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This well-known fact will, of course, be confirmed by the present paper, but conceptually the situation is not always obvious at first sight. The following example will certainly strike us as paradoxical: The most accurate means for practically establishing an inertial system is VLBI using quasars; however, quasars are a typical phenomenon of an expanding universe which must be described by general relativity and for which, therefore, rigorously no inertial systems exist!

We shall come back to this paradox later on. It already indicates that an understanding of the basic principles of reference systems from a relativistic point of view is of conceptual significance.

The present paper attempts a review of the impact of relativity on the problem of reference systems. We shall first discuss the concept of inertial system from the point of view of both the special and the general theory of relativity, then give a review of relativistic corrections and similar effects by which relativistic geometry and mechanics differ from the classical situation, and finally discuss the problem of a possible variation of the gravitational constant G which, however, goes beyond Einsteinian relativity.

There are a number of excellent textbooks on the theory of relativity. The most elegant presentation is perhaps (Synge, 1960, 1972), the most comprehensive and modern text is certainly (Misner et al., 1973), and a very readable and useful recent book is (Ohanian, 1976). An excellent review article on applications to space science is (Dicke and Peebles, 1965). In a previous work (Moritz, 1967), the author has treated in some detail the question of inertial systems from a relativistic standpoint, especially with a view to separation of gravitation and inertia which is not usually considered in standard textbooks (except Synge, 1960). The present paper partly follows (Moritz, 1979).

Like all great and deep theories, Einstein's theory admits of various, often controversial, interpretations. It has even been argued that the name, general relativity, is not entirely appropriate since the essence of this theory is not the general "relativity" of all reference systems but rather the fact that the theory provides a mathematical description of "absolute" curved space-time. This point of view seems to be rather widely accepted at present (e.g. Fock, 1959 ; Synge, 1960 ; Misner et al., 1973 ; Ohanian, 1976); it is also favored in the present article.

2. INERTIAL SYSTEMS AND RELATIVITY

Inertial systems in special relativity. In the special the-

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ory of relativity, inertial systems play a basic role as privileged coordinate systems in space-time: in such a system, the four-dimensional line element has the simple form

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 \; . \quad (1)$$

Here $x = x_1$, $y = x_2$, and $z = x_3$ denote rectangular coordinates in space, t designates the time, and c denotes the constant velocity of light in a vacuum; we have to put $x_4 = ict$, where $i^2 = -1$. As in classical mechanics, a reference system moving with constant velocity with respect to an inertial system, is again an inertial system.

Transformations between inertial systems are such as to leave the line element (1) invariant (unchanged); the set of such "Lorentz transformations" form a group, the Lorentz group, which describes the symmetry of the space-time of special relativity.

No inertial systems in general relativity. The special theory of relativity holds only in the absence of a gravitational field. Gravitational fields are treated by the general theory of relativity. Here the line element has the form

$$ds^2 = \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} g_{\alpha\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta \quad (2)$$

where x^α denotes coordinates x^1, x^2, x^3, x^4 in space-time, which will in general be curvilinear rather than rectangular. The $g_{\alpha\beta}$ are functions of these coordinates. Indices such as α and β run from 1 to 4 ; lower indices are called covariant, and upper indices, contravariant. The Einstein summation convention, which will be used in this section, prescribes summation with respect to any index that occurs in both an upper and a lower position, as shown in eq. (2). The coordinates x^α now have upper indices because the differentials dx^α form a "contravariant vector".

The line element (2) relates to (1) in much the same way as a line element on a curved surface,

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2, \quad (3)$$

relates to a line element in the plane,

$$ds^2 = dx^2 + dy^2. \quad (4)$$

Here, u, v are curvilinear coordinates and E, F, G form the "metric tensor"

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$$\begin{bmatrix} E & F \\ F & G \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} . \tag{5}$$

In space-time, the metric tensor $[g_{\alpha\beta}]$ is a 4x4 matrix, so that there is full analogy between the general forms (2) and (3) on the one hand, and between the "inertial forms" (1) and (4) on the other hand. In a way, the general theory of a relativity is nothing else but an extension of the theory of two-dimensional surfaces to four-dimensional space-time.

This analogy will help understand an important point. On a curved surface one can introduce coordinates which, in an infinitesimal neighborhood of a point, give a line element

$$ds^2 = du^2 + dv^2 \tag{6}$$

which has the same form as the plane element (4) . Geometrically, this means that, in a small neighborhood of this point, the surface is approximated by its tangent plane. However, it will not be possible, in general, to introduce coordinates in such a way that the "inertial form" (6) holds on the whole surface (or even in a finite part of it).

Transferred to four dimensions, this reasoning shows that, in a curved space-time, it will be possible to introduce coordinates which correspond to an inertial system in an infinitesimal neighborhood of a point; but it is not possible to introduce an inertial system that is valid for the whole space-time.

In this sense, there are no inertial systems in general relativity. All possible coordinate systems are, in principle, equivalent; there are no privileged systems. This is Einstein's Principle of General Covariance, or General Relativity.

Another important principle in this theory is the Principle of Equivalence, according to which gravitational and inertial forces (such as the centrifugal or Coriolis force) are basically identical: both are effects of a deviation of the coordinate system of line element (2) from an inertial system of line element (1) . Thus gravitation is interpreted geometrically as an effect of the curvature of space-time.

Both the Principle of Equivalence and the Principle of General Covariance have played a fundamental heuristic role in Einstein's considerations leading to his theory of gravitation around 1915 because these principles provide a natural transition from the flat space-time of special relativity to the curved space-time of general relativity. Einstein's heuristic procedure is still the

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best way for understanding this theory; hence it is strongly emphasized in almost every textbook on general relativity.

The relativistic treatment of reference systems, however, requires some subtler distinctions which show that, after all, privileged systems can be introduced which serve as practically satisfactory approximations to inertial systems, both on a local and on a global level.

Local inertial systems. Just as a curved surface can be approximated locally by a tangent plane, so curved space-time can be approximated, in the neighborhood of a certain point, by a tangent "plane" space-time in which an inertial system can be introduced. Thus, in a certain "small" region, inertial systems are possible even in general relativity. Since our space-time is only very slightly curved, the gravitational field in the solar system being very weak, the "small" region just mentioned certainly covers the solar system and even extends well beyond. According to Eddington (1924, pp.99) a local inertial system will deviate from a global system by about 2 seconds of arc in a century.

Global nearly-inertial systems. The application of the relativistic theory of gravitation to the region of our solar system requires boundary conditions at infinity: with increasing distance from the attracting masses the effect of gravitation vanishes, and the curved space-time becomes flat at infinity. This fact permits the introduction of uniquely defined privileged systems, the harmonic coordinate systems. These systems rigorously refer to curved space-time. At infinity they reduce to inertial systems of form (1) , and within the solar system they approximate inertial systems practically very well.

In this sense, the harmonic coordinates form a privileged coordinate system, which is a natural generalization of an inertial system to curved space-time. This has been particularly emphazised by Fock (1959); see also (Weinberg, 1972, p. 162).

Quasi-inertial systems and Fermi propagation. Let us introduce the concept of quasi-inertial systems. They are three-dimensional cartesian systems whose origin is moving arbitrarily but whose axes remain always parallel; a physical realization is by means of axes whose direction is stabilized by means of gyroscopes. The underlying principle is that the axis of a freely spinning gyroscope maintains its direction even if its frame is accelerated or rotated; furthermore, the axis is unaffected by gravity.

Quasi-inertial systems differ from inertial systems in the strict sense by the fact that they can be in nonuniform (accelerated) motion with respect to each other, as long as the coordinate axes remain parallel. Inertial systems are always in uniform motion, that is, they move with a constant velocity vector with respect to each other. A geocentric system of which the axes have a constant direction in space is an example of a quasi-inertial system: the origin (the geocenter) moves along an ellipse around the sun, rather than along a straight line with constant velocity.

This concept of a quasi-inertial frame can be defined also in general relativity. The relevant concept is Fermi propagation, or Fermi-Walker transport, which is considered in detail and used extensively in (Synge, 1960). It is also treated in (Misner et al., 1973, p. 170), but hardly elsewhere in standard textbooks. Therefore we shall briefly consider it here, following (Moritz, 1967).

The equation of Fermi-Walker transport may be written

$$\frac{\delta\lambda^\alpha}{\delta s} = \lambda_\beta \left(\frac{\delta u^\beta}{\delta s} u^\alpha - \frac{\delta u^\alpha}{\delta s} u^\beta\right) \tag{7}$$

(Synge, 1960, p. 13). Here λ^α (or λ_β) are the contravariant (or covariant) components of the vector undergoing Fermi propagation, related by

$$\lambda_\alpha = g_{\alpha\beta}\lambda^\beta . \tag{8}$$

The vector u^α is the four-velocity

$$u^\alpha = \frac{dx^\alpha}{ds} , \tag{9}$$

the unit vector of the tangent to the world line of the particle to which the vector λ_α is attached. The symbol δ denotes covariant differentiation:

$$\frac{\delta\lambda^\alpha}{\delta s} = \frac{d\lambda^\alpha}{ds} + \Gamma^\alpha_{\beta\gamma}\lambda^\beta u^\gamma , \tag{10}$$

where $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols, and analogously for $\delta u^\alpha/\delta s$.

In our case, the vector λ^α represents the spin axis of the gyroscope. It lies in the instantaneous three-dimensional space of the spinning particle and is therefore orthogonal to u^α :

$$u^\alpha\lambda_\alpha = 0 . \tag{11}$$

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Hence (7) reduces to

$$\frac{\delta\lambda^\alpha}{\delta s} = \lambda_\beta \frac{\delta u^\beta}{\delta s} u^\alpha . \qquad (12)$$

This equation holds for Fermi-Walker transport of a space-like vector satisfying (11) . It expresses the fact that the change $\delta\lambda^\alpha/\delta s$ has the direction of u^α and has no component in the instantaneous three-space of the observer. Thus the change λ^α is purely in time: the vector λ^α remains unchanged in space, it is transported parallelly. This shows that Fermi propagation is related to spatial parallelism.

Consider now a system of three mutually orthogonal space-like vectors λ^α , each of which is represented by the axis of a freely spinning gyroscope. In this way the axes of a rectangular xyz system which is transported parallelly in space, may be realized physically.

It can be shown (Moritz, 1967, p. 47) that the change $\delta\lambda^\alpha/\delta s$ is small of order c^{-2} , c being the velocity of light. To this accuracy, the direction of Fermi-propagated axes remains constant in space; it furthermore is practically unaffected by the gravitational field.

This shows that gyroscopically stabilized "quasi-inertial systems" are possible even in the context of general relativity.

Separation of gravitation and inertia. After this discussion of "privileged" coordinate systems which seem to contradict the Principle of General Covariance, let us now briefly remark on the separation of gravitational and inertial forces, which seems to violate the Principle of Equivalence. This question is related to the problem of reference systems only indirectly; it has been dealt with rather fully in two reports (Moritz, 1967, 1971).

The Principle of Equivalence states that, because of the identity of gravitational and inertial mass (shown experimentally by R. Eötvös around 1900 to an accuracy of 5×10^{-9} !) the resultant of gravitational and inertial forces acting at one point cannot be separated into a gravitational and an inertial part; both are equivalent and cannot be distinguished.

Matters are different if we consider, not only one point, but a region in space, which may be arbitrarily small. In the theory of surfaces, the Gaussian curvature K provides a criterion for distinguishing a curved surface from a plane, depending on whether K is nonzero or zero. The generalization of the Gaussian curvature to four dimensions is the Riemannian curvature tensor

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$R_{\alpha\beta\gamma\delta}$; again, space-time is flat if $R_{\alpha\beta\gamma\delta} = 0$ and curved otherwise. Now, curvature of space-time is an objective criterion for the presence of a genuine gravitational field, so that, according to (Synge, 1960, p. 109), we may write

$$R_{\alpha\beta\gamma\delta} = \text{gravitational field} . \tag{13}$$

The Riemann curvature thus provides a criterion for the presence of a gravitational field, but not yet a means for the separation of gravitational and inertial effects. In flat space-time, inertial forces have an objective significance since they are due to the deviation of the observer's coordinate system from an inertial system. Similarly in a weak gravitational field, a separation of gravitation and inertia is feasible if we succeed in introducing a privileged coordinate system similar to an inertial system. In this way, the separation of gravitation and inertia is intimately connected with the question of an "almost" inertial reference system, such as the harmonic system mentioned above.

We finally point out that in such a system there is approximately (Moritz, 1967, p. 43)

$$c^2 R_{i4j4} = \frac{\partial^2 V}{\partial x_i \partial x_j} \tag{14}$$

where i and j are spatial indices running from 1 to 3 . Thus, second-order gradients of the potential V are purely gravitational. In (Moritz, 1971) we have shown that using a combination of accelerometers, measuring first-order gradients, and gradiometers, measuring second-order gradients, a separation of the gravitational signal from inertial disturbances can be effected even with first-order gradients, that is, in the gravitational force.

Cosmological questions. For a homogeneous and isotropic universe, the line element (2) has the form (Bondi, 1960, p. 102)

$$ds^2 = dt^2 - [R(t)]^2 \frac{dx^2 + dy^2 + dz^2}{[1 + (k/4)(x^2 + y^2 + z^2)]^2} . \tag{15}$$

Here $R(t)$ is a time-dependent scale factor by means of which the expansion of the universe can be described. The constant k may have the values +1, 0, or -1 . For $k = 0$, space is Euclidean; for $k = 1$, space has constant positive curvature, and for $k = -1$, constant negative curvature. The space-time described by (15) is called the Robertson-Walker model. (For $k = 0$ and $R = c^{-1}$, (15) reduces to (1) , apart from the irrelevant factor $(-c^2)$.)

This model appears well suited to describe mathematically the large-scale space-time structure of the universe, apart, of course, from "local" gravitational irregularities such as caused by our solar system. On the basis of present observational data it is not possible to decide clearly whether k is positive, negative or zero, although there is some indication that space may have negative curvature (cf. Ohanian, 1976, p. 416).

At any rate, Robertson-Walker space-time will not in general be the flat space-time of special relativity (1) . Thus, strictly speaking, inertial systems in the usual sense will not exist. This leads us to the paradox already mentioned in the introduction: The most accurate means of practically establishing an inertial system is VLBI using quasars; however, quasars are a typical phenomenon of an expanding universe which is described by the curved space-time (15) for which no inertial system exists!

This paradox, however, is a theoretical curiosity rather than a fact of particular significance. Indeed, as we have seen above, all our practical inertial systems are nonrigorous in the sense of general relativity but still perfectly useful. For the region of our galaxy, we may easily consider space-time to be essentially flat, apart from local gravitational irregularities. The same holds a forteriori for our solar system. Furthermore, it is possible to study cosmology within the frame of special relativity and even of classical mechanics (Bondi, 1960, Chapters XI and IX).

3. RELATIVISTIC CORRECTIONS

The mathematical description of geometry and gravitational field around the Earth (geodesy, geodynamics, satellite dynamics) and in the solar system (celestial mechanics, classical astronomy) uses Euclidean geometry and classical mechanics. Such a description is valid to an accuracy of about 1 part in 10^8 . For higher precisions, the special and general theories of relativity must be taken into account. This is best done by applying small "relativistic corrections" to the classical formulas.

Post-Newtonian approximation. Let us formulate the equations of general relativity in an approximate form which is sufficiently accurate for one purpose and, at the same time, comes close to classical potential theory. Such "nearly-Newtonian gravitational fields" or post-Newtonian approximations" to Einstein's theory are treated in almost every text on relativity; cf. (Misner et al., 1973, pp. 445, 1068) and (Boucher, 1979).

For this case the general line element reduces to

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$$ds^2 = (1 + 2 \frac{V}{c^2})(dx^2 + dy^2 + dz^2) - (1 - 2 \frac{V}{c^2}) c^2dt^2 . \quad (16)$$

Here x, y, z are rectangular spatial coordinates as usual, and t is a time coordinate; it will be called coordinate time. The symbol V denotes the classical Newtonian gravitational potential, defined in the "geodetic" way (everywhere positive and tending to zero at infinity; physicists frequently use the opposite sign) and c is the velocity of light as usual. This equation is linear in V/c^2 ; higher powers are consistently neglected in this theory.

What is the order of magnitude of V/c^2 ? The gravity potential W at the surface of the earth is approximately

$$W = 6.3 \times 10^7 m^2 s^{-2} ;$$

cf. the value U_o given in (Heiskanen and Moritz, 1967, p. 80); for the present purpose, the gravitational potential V and the gravity potential W (including the centrifugal force) are nearly equal. Then

$$\frac{V}{c^2} \doteq \frac{W}{c^2} \doteq \frac{6.3 \times 10^7 m^2 s^{-2}}{(3 \times 10^8 ms^{-1})^2} \doteq 0.7 \times 10^{-9} . \quad (17)$$

Thus, V/c^2 is a dimensionless quantity of order 10^{-9} at the earth's surface (and smaller at higher elevations). If we neglect this small quantity, then the line element (16) reduces to the simple line element (1)) of special relativity.

Time. Since time can be measured by means of atomic clocks far more accurately (to order 10^{-13} or better) than any other relevant quantity, relativistic effects show here quite well and must be taken into account. Atomic time which an atomic clock measures, has the character of a proper time and will be denoted by τ . The element of proper time, $d\tau$, is proportional to the element ds , given by (16) , of the world line of the atomic clock:

$$ds = icd\tau , \qquad d\tau = ds/ic . \quad (18)$$

Hence, for a clock at rest $(dx = dy = dx = 0)$,

$$d\tau = (1 - 2 \frac{V}{c^2})^{\frac{1}{2}} dt \doteq (1 - \frac{V}{c^2}) dt . \quad (19)$$

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If the atomic clock is fixed to the rotating Earth, then the gravitational potential V in (19) must be replaced by the gravity potential W which is the sum of V and of the centrifugal potential. Thus atomic clocks depend on the potential in a similar way as the old pendulum clocks depend on gravity, through incomparably less. Just as gravity, or better gravity differences, can be measured by means of pendulums, so the potential, or better potential differences, can, in principle, be measured by atomic clocks. (It would, however, be premature to hope for a new geodetic instrument measuring potential differences in this way: 1 cm in elevation would correspond to 10^{-18} in time!)

Of such nature is the experiment by Pound and Rebka described in (Misner et al., 1973, p. 1057) and (Ohanian, 1976, p.212), which measures the gravitational redshift of γ-rays using the Mössbauer effect and hence the potential difference. (Redshift occurs if the "clock" represented by the emitting source is slower.)

Related phenomena are the time delay of radar echoes from Mercury, Venus and Mars due to their gravitational fields as measured by Shapiro and others (Ohanian, 1976, p. 128), and time dilation experiments measuring the redshift of different spectral lines of the sun and other stars. (Ohanian, 1976,p.214). We shall consider such an effect below when discussing laser distance measurements.

Another question is the relation between Atomic Time (AT) and Ephemeris Time (ET). Conceptually, AT is the time of quantum theory, and ET is the time of mechanics (classical or relativistic). If general relativity is correct, then AT = ET. On the other hand, (Duncombe et al., 1974, p. 232) state that empirical observations tend to indicate that these two time scales are not equivalent. As an explanation they suggest that the gravitational constant G decreases at the rate of about 10^{-11} per year. We shall consider the question of temporal variability of G in sec. 4. For the time being, however, we shall limit ourselves to general relativity in the Einsteinian sense, for which G is constant.

As a practical consequence we note that eq. (19) can be used to reduce atomic time τ to coordinate time t. A more general expression is the well known formula

$$\frac{d\tau}{dt} = 1 - \frac{v^2}{2c^2} - \frac{V}{c^2} , \qquad (20)$$

which is an immediate consequence of (16) and (18), putting $dx^2 + dy^2 + dz^2 = v^2 dt^2$ and neglecting the term Vv^2/c^4 as being of higher order. Here v is the velocity of the clock in the basic system $xyzt$.

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The term v^2/c^2 is a special-relativistic correction, and V/c^2 represents a general-relativistic contribution. The orders of magnitude of these corrections are as follows:

$\frac{v^2}{c^2} \doteq 10^{-8}$ for the Earth's orbital speed (30 km/sec)

$\frac{v^2}{c^2} \doteq 10^{-12}$ for the Earth's rotational speed (0.46 km/sec at the equator)

$\frac{V}{c^2} \doteq 10^{-8}$ for the Sun's gravitational potential at the Earth's orbit

$\frac{V}{c^2} \doteq 10^{-9}$ for the geopotential at the Earth's surface; cf. also (17).

The reduction from atomic time to coordinate time is thus given by

$$t = \int \left(1 + \frac{v^2}{2c^2} + \frac{V}{c^2}\right) d\tau \; . \tag{21}$$

This reduction permits us to get a uniform time scale which is not affected by motion and by gravitational irregularities (Thomas, 1975).

Length. The present definition of the meter in terms of a certain multiple of the orange line of krypton will probably be given up in the near future. It will be redefined in terms of the atomic second and the velocity of light, of which the present accepted value is

$$c = (299\ 792\ 458 \pm 1.2)\,\mathrm{ms}^{-1} \tag{22}$$

(Moritz, 1975). This indirect definition of length will be more accurate.

In fact, since c is accurate to about 4 parts in 10^9, the new definition of length will be as accurate (time being defined with superior precision). Relativistic effects are below this level, so that the influence of these effects on length will be negligible still for some time.

Doppler measurement. Doppler shift (changes in frequency) and aberration (changes of direction) of light or of another electromagnetic wave are fundamental phenomena in special relativity and are treated in almost all textbooks on relativity (for an

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astronomical presentation cf. Schneider, 1979, ch. 11). Relativistic Doppler shift and aberration differ from their classical counterparts by the factor $(1-v^2/c^2)$, which is very close to unity.

If λ is the wave length as omitted by the source, λ' is the received wave length, and $\Delta\lambda = \lambda' - \lambda$, then

$$\frac{\Delta\lambda}{\lambda} = (1 + \frac{v_r}{c})(1 - \frac{v^2}{c^2})^{-\frac{1}{2}} - 1 \doteq \frac{v_r}{c} + \frac{v^2}{2c^2} . \tag{23}$$

Here v_r is the radial component of the velocity vector $\underline{v}$ of the source with respect to the observer, and v is the norm of $\underline{v}$. Eq. (23) differs from the classical Doppler shift v_r/c by the "second order Doppler correction" $v^2/2c^2$, which is a special-relativistic effect. The latter is present even if the velocity vector $\underline{v}$ has no radial component v_r. Therefore $v^2/2c^2$ is also called "transversal Doppler effect"; it is due to the apparent retardation of the moving clock (the source).

If one considers also the effect of gravitation on the clock frequency by (20), then (23) becomes

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} + \frac{v^2}{2c^2} + \frac{V_S - V_R}{c^2} , \tag{24}$$

where $V_S - V_R$ is the potential difference between sender (source, clock) S and receiver R.

If the sender is in a satellite and the receiver is at the Earth's surface, then (24) applies to geodetic Doppler observations. The second-order corrections (second and last term on the right-hand side) cancel partly since $V_R > V_S$. For normal satellite heights, the general-relativistic correction is smaller by an order of magnitude; however, if the satellite height reaches half of the Earth's radius, then the two second-order corrections cancel completely (Weinberg, 1972, p. 84). For a satellite height of 1000 km, the second-order Doppler correction gives $\Delta\lambda/\lambda \doteq v^2/2c^2 \doteq 3 \times 10^{-10}$. It is also worth noting that in satellite Doppler positioning, second-order Doppler effects (from both special and general relativity) can be treated as constant frequency bias (Blais, 1977; Boucher, 1976, 1978).

Laser distance measurements. The velocity of light in the presence of a gravitational field is not c but $v = c(1 - 2V/c^2)$; this is an immediate consequence of (16) on putting $ds = 0$ for light and $dx^2 + dy^2 + dz^2 = v^2dt^2$. Thus the distance s computed by multiplying the travel time by c must be diminished by

$$\delta s = \frac{2}{c} \int_P^S V dt .$$

Here P is the laser station and S is the reflecting satellite. To a sufficient accuracy we may put $V = GM/r$ and integrate simply from $r = R$ to $r = R + H$, where R is the Earth's radius and H is the satellite elevation, putting $dt = dr/c$. Thus

$$\delta s = \frac{2GM}{c^2} \int_R^{R+H} \frac{dr}{r} = \frac{2GM}{c^2} \ln \frac{R+h}{R} , \qquad (25)$$

which reaches 1 mm for H = 1000 km and 4 cm for the moon. (For lunar laser ranging cf. Mulholland, 1977; Stolz, 1979.)

Very-long- baseline interferometry. Here it is customary to reduce observed atomic time to coordinate time with respect to an inertial system with origin at the center of the solar system. The reduction formula, obtained by an appropriate evaluation of (21), has the principal term (Thomas, 1975; Robertson, 1975b)

$$\Delta t = -\frac{1}{c^2} \underline{v} \cdot \underline{x} , \qquad (26)$$

where $\underline{v}$ is the Earth's orbital speed and $\underline{x}$ is the clock's geocentric position vector. This term has a daily period and an amplitude of about 1.5 μs ; it can also be explained as a classical aberration effect. For a detailed discussion see (Thomas, 1975). Aberration effects in VLBI are considered from the standpoint of special relativity in (Robertson, 1975a).

Deflection of light. Light rays can be regarded as straight except under unusual circumstances. Classical is the deviation of a light ray grazing the sun during an occultation. Modern results concerning this phenomenon and concerning analogous deflections of radio waves are given in (Ohanian, 1976, pp. 124-125); the order of magnitude is 1 - 2 seconds of arc.

Gyroscopic effects. Above we have seen that gyroscopes undergoing Fermi-Walker transport behave very much as in classical mechanics. Small relativistic effects ("geodetic precession") are described in (Ohanian, 1976, pp. 292-298).

Influence on planetary motion. The classical example is the precession of the perihelion of the orbit of the planet Mercury (about 40" per century). There are also periodic relativistic effects in earth-moon separation on the order of 1 m , which can be measured by lunar lasar ranging (Misner et al., 1973, p. 1048).

(For relativistic effects on satellite orbits, cf. Rubincam, 1977.)

4. IS THE GRAVITATIONAL CONSTANT CONSTANT?

As an explanation of certain astronomical and geophysical phenomena, it has been suggested (Dicke, 1964; Dicke and Peebles, 1965; Duncombe et al., 1974) that the gravitational constant G is decreasing by a few parts in 10^{11} per year. The evidence is not clear, however; it seems to be difficult to separate a true change of G from other systematic influences (Ohanian, 1976, p. 188; Stephenson, 1978). The evidence on which the conclusions of Dicke and Peebles (1965) are based, is now superseded by recent data on the secular variation of the earth's rotation, summarized in (Lambeck, 1980, pp. 299-319). An experimental bound, $|\dot{G}/G| < 4 \times 10^{-10}$ has been obtained by Shapiro et al. (1971) by analyzing radar-echo time delays between Earth and Mercury (see also Williams et al., 1978).

In Einstein's general theory of relativity, G is constant. A changing G requires a different theory; such theories have been proposed by Jordan, Brans, and Dicke (Misner et al., 1973, p.1070) and Treder (1977). Since Einstein's theory is of incomparable simplicity and perfection, most physicists would be willing to give it up only in the presence of very solid empirical evidence. For the purposes of reference systems and time scales it thus appears permissible at present to take a conservative attitude and remain within the frame of Einstein's theory.

REFERENCES

Blais, J.A.R.: 1977, *Inertia, Inertial Reference Systems and Physical Geodesy*, Am. Geophys. Union Fall Meeting, San Francisco, Calif.

Bondi, H.: 1960, *Cosmology*, Cambridge Univ. Press.

Boucher, C.: 1976, *Modelisation des mesures Doppler effectuées par le récepteur JMR-1 sur le système Transit*, No. 26748, Inst. Géogr. Nation., Paris.

Boucher, C.: 1978, *Relativistic Correction to Satellite Doppler Observation*, Meeting of IAG SSG 4.45, Mathematical Structure of the Gravity Field, Lagonissi (Greece).

Boucher, C.: 1979, *The Relativistic Representation of the Gravity Field*, IUGG/IAG General Assembly, Canberra.

Dicke, R.H.: 1964, in De Witt and De Witt (eds.), *Relativity, Groups and Topology*, Gordon and Breach, New York, pp.163-313.

Dicke, R.H., Peebles, P.J.: 1965, Space Sci.Rev. 4, pp. 419-460.

Duncombe, R.L., Seidelmann, P.K., Van Flandern, T.C.: 1974, IAU Coll. 26, Torún, pp. 223-233.

Eddington, A.S.: 1924, *The Mathematical Theory of Relativity*, Cambridge University Press.

https://doi.org/10.1017/S0252921100081094 Published online by Cambridge University Press

Fock, V.: 1959, *The Theory of Space Time and Gravitation*, Pergamon Press, London.
Heiskanen, W.A., Moritz, H.: 1967, *Physical Geodesy*, W.H. Freeman, San Francisco.
Lambeck, K.: 1980, *The Earth's Variable Rotation*, Cambridge University Press.
Misner, C.W., Thorne, K.S., Wheeler, J.A.: 1973, *Gravitation*, W.H. Freeman, San Francisco.
Moritz, H.: 1967, Report No. 92, Dept. of Geodet. Sci., Ohio State Univ.
Moritz, H.: 1971, Report No. 165, Dept. of Geodet. Sci, Ohio State Univ.
Moritz, H.: 1975, Bull. Géod. 118, pp. 398-408.
Moritz, H.: 1979, Report No. 294, Dept. of Geodet. Sci, Ohio State Univ.
Mulholland, J.D. (ed.): 1977, *Scientific Applications of Lunar Laser Ranging*, D. Reidel, Dordrecht, Holland (articles by J.D. Mulholland and J.G. Williams).
Ohanian, H.C.: 1976, *Gravitation and Spacetime*, W.W. Norton, New York.
Robertson, D.S.: 1975a, Nature 257, pp. 467-468.
Robertson, D.S.: 1975b, Report X-922-77-228, Goddard Space Flight Center, Greenbelt, Md.
Rubincam, D.P.: 1977, Celestial Mechanics 15, pp. 21-33.
Schneider, M.: 1979, *Himmelsmechanik*, Bibliographisches Institut, Zürich.
Shapiro, I.I., Smith, W.B., Ash, M.B., Ingalls, R.P., Pettengill, G.H.: 1971, Phys. Rev. Lett. 26, pp. 27-30.
Stephenson, F.R.: 1978, in P. Brosche and J. Sündermann (eds)., *Tidal Friction and the Earth's Rotation*, Springer, Berlin, pp. 5-21.
Stolz, A.: 1979, Deutsche Geodätische Kommission, A,90, München.
Synge, J.L.: 1960, *Relativity: The General Theory*, North Holland Publ. Co., Amsterdam.
Synge, J.L.: 1972, *Relativity: The Special Theory*, North Holland Publ. Co., Amsterdam.
Thomas, J.B.: 1975, Astron. J. 80, pp. 405-411.
Treder, H.J.: 1977, in *Third International Symposium, Geodesy and Physics of the Earth*, Veröffentlichungen des Zentralinstituts für Physik der Erde, 52(1), pp.119-127, Potsdam.
Weinberg, S.: 1972, *Gravitation and Cosmology*, Wiley, New York.
Williams, J.G., Sinclair, W.S., and Yoder, C.F.: 1978, Geophys. Res. Lett. 5, pp. 943-946.