

**This text provides a self-contained introduction to applications of loop representations and knot theory in particle physics and quantum gravity.**

**Loop representations (and the related topic of knot theory) are of considerable current interest because they provide a unified arena for the study of the gauge invariant quantization of Yang-Mills theories and gravity, and suggest a promising approach to the eventual unification of the four fundamental forces. This text begins with a review of calculus in loop space and the fundamentals of loop representations. It then goes on to describe loop representations in Maxwell theory and Yang-Mills theories as well as lattice techniques. Applications in quantum gravity are then discussed in detail. Following chapters move on to consider knot theories, the braid algebra and extended loop representations in quantum gravity. A final chapter assesses the current status of the theory and points out possible directions for future research.**

**This self-contained introduction will be of interest to graduate students and researchers in theoretical physics and applied mathematics.**

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# Loops, Knots, Gauge Theories and Quantum Gravity

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