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THE CHARACTER TABLES FOR SL(3, q), SU(3, q^2), PSL(3, q), PSU(3, q^2)

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1. Introduction. In this paper the character table of GL(3, q) (U(3, q^2)), the group of all nonsingular $n \times n$ (unitary) matrices over GF(q) (GF(q^2)), is used to obtain the character tables for the related subgroups SL(3, q), PSL(3, q) (SU(3, q^2), PSU(3, q^2)), the corresponding groups of matrices of determinant unity and the projective group respectively. There are very few abstract character tables which hold for entire families of groups. Such tables are of much greater value than tables for specific groups because, among other things, they enable one to discern various patterns common to the whole family. The families of groups mentioned in the title are of particular interest because they frequently occur as important subgroups of simple groups. In fact, the families PSL(3, q), PSU(3, q^2) are nearly all simple groups. In this paper we restrict our efforts to the case $n = 3$. The case of $n = 2$ has already been done by several researchers; PSL(2, q) $\cong$ PSU(2, q^2) by Jordan [**6**], SL(2, q) $\cong$ SU(2, q^2) by I. Schur [**7**]. In 1924, Brinkmann obtained the character tables for GL(3, q), SL(3, q), PSL(3, q); however, this unpublished paper is not available and for this reason these groups are again considered here. Although the methods used are completely general and in no way depend upon n being a particular value, the practicality of using this technique for $n = 4$ or larger is questionable. The reasons for this will be taken up in § 6.

Steinberg [**8**] determined the character table for GL(3, q) and Ennola [**2**] found the table for U(3, q^2). Using these two tables and the extremely explicit notation used by Ennola we employ the general method of restriction to obtain the tables for the above mentioned groups. The results of the paper are three-fold:

(1) The tables for SL, SU, PSL, PSU for $n = 3$ are given in § 7 in a very compact and explicit form which facilitates their use.

(2) A conjecture made by Ennola [**2**] relating the tables of the unitary and general linear groups is extended in § 5 to the subgroups under discussion. This conjecture is verified for $n = 2, 3$ and is used to combine the character tables for the corresponding linear and unitary groups.

(3) A stronger version of Clifford's theorem is proved in § 4 which although applicable only to the groups SL(n, q) and SU(n, q^2) is nevertheless very useful in the development of their character tables.

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2. Procedure for obtaining the characters of SL(3, q), $d = 1, 3$. Only the linear groups will be discussed throughout the paper; however, due to the close parallelism of the linear and unitary groups, a replacement of $(q - 1)$ by $(q + 1)$ throughout the discussion will furnish one with the procedure followed for the unitary groups.

In [**8**] the character table for GL(3, q) is given in two tables. The first table shows the conjugacy class structure. The classes are represented by a corresponding Jordan canonical form over a suitable extension field. (In the unitary case the classes can be indexed by using certain Jordan canonical forms for matrices in GL(3, q^2), as shown in [**9**] and [**3**].) Classes having the same type canonical representative are grouped together and indexed by the letters k, l, m. In the second table the characters of the group are given. They also are arranged in sets, each set containing all the characters of the same degree; the characters in each set are indexed by the letters u, v, w. We first determine the conjugacy class structure of SL(3, q) by setting the conditions on the exponents k, l, m of the canonical representatives which are necessary in order for the determinant to be unity. The number of conjugacy classes of SL

TABLE 1a

Conjugacy Class Structure for SL(3, q), SU(3, q^2), $d = 1, 3$

Conjugacy class	Canonical representative	Parameters	Number of classes	Centralizer order
$C_1^{(k)}$	$\begin{bmatrix} \omega^k & & \\ & \omega^k & \\ & & \omega^k \end{bmatrix}$	$0 \leqq k \leqq (d-1)$	d	q^3r^2st
$C_2^{(k)}$	$\begin{bmatrix} \omega^k & & \\ 1 & \omega^k & \\ & & \omega^k \end{bmatrix}$	$0 \leqq k \leqq (d-1)$	d	q^3r
$C_3^{(k,l)}$	$\begin{bmatrix} \omega^k & & \\ \theta^l & \omega^k & \\ & \theta^l & \omega^k \end{bmatrix}$	$0 \leqq k, l \leqq (d-1)$	d^2	dq^2
$C_4^{(k)}$	$\begin{bmatrix} \rho^k & & \\ & \rho^k & \\ & & \rho^{-2k} \end{bmatrix}$	$1 \leqq k < r$ $k \not\equiv 0 \pmod{r'}$	$r - d$	qr^2s
$C_5^{(k)}$	$\begin{bmatrix} \rho^k & & \\ 1 & \rho^k & \\ & & \rho^{-2k} \end{bmatrix}$	$1 \leqq k < r$ $k \not\equiv 0 \pmod{r'}$	$r - d$	qr
$C_6^{(k,l,m)}$	$\begin{bmatrix} \rho^k & & \\ & \rho^l & \\ & & \rho^m \end{bmatrix}$	$1 \leqq k, l, m \leqq r$ $k < l < m$ $k + l + m \equiv 0 \pmod{r}$	$\frac{1}{6}r(r-3) + \frac{1}{3}d$	r^2
$C_7^{(k)}$	$\begin{bmatrix} \rho^k & & \\ & \sigma^{-\delta k} & \\ & & \sigma^{-qk} \end{bmatrix}$	$1 \leqq k < rs$ $k \not\equiv 0 \pmod{s}$ $C^{(k)} = C^{(\delta kq)} \pmod{rs}$	$\frac{1}{2}(s-1)r$	rs
$C_8^{(k)}$	$\begin{bmatrix} \tau^k & & \\ & \tau^{kq} & \\ & & \tau^{kq^2} \end{bmatrix}$	$1 \leqq k < t$ $k \not\equiv 0 \pmod{t'}$ $C^{(k)} = C^{(kq)} = C^{(kq^2)} \pmod{t}$	$\frac{1}{3}(t-d)$	t

TABLE 1b.

Character table for SL(3, q), SU(3, q^2) $d = 1, 3$

Parameters				$1 \leqq u \leqq (r-1)$		$1 \leqq u < v \leqq r'$; $v < w \leqq r$; $u + v + w = 0 \pmod r$	$0 \leqq u \leqq d - d' - 1$	$1 \leqq u \leqq rs$; $u \not\equiv 0 \pmod s$; $(u) = (\delta u q) \pmod{rs}$	$1 \leqq u \leqq t$; $u \not\equiv 0 \pmod{t'}$; $(u) = (uq\delta) = (uq^2) \pmod{t'}$	$0 \leqq u \leqq d - d' - 1$	
Number of characters	1	1	1	$r-1$	$r-1$	$\frac{1}{6}[r(r-3) + (3-d)]$	$d - d'$	$\frac{1}{2}(s-1)r$	$\frac{1}{3}(t-d)$	$d - d'$	$d - d'$
class \ char.	χ_1	χ_{qs}	χ_{q^3}	$\chi_t^{(u)}$	$\chi_{qt}^{(u)}$	$\chi_{st}^{(u,v,w)}$	$\chi_{st'}^{(u)}$	$\chi_{rt}^{(u)}$	$\chi_{r^2s}^{(u)}$	$\chi_{r^2s'}^{(u)}$	$\chi_{r^2s'}^{(u)'}$
$C_1^{(k)}$	1	qs	q^3	$t\omega^{uk}$	$qt\omega^{uk}$	$st\omega^{(u+v+w)k}$	st'	$rt\omega^{uk}$	$r^2s\omega^{uk}$	$r^2s'\omega^k$	$r^2s'\omega^{2k}$
$C_2^{(k)}$	1	δq	0	$\delta s\omega^{uk}$	$q\omega^{uk}$	$(2q+\delta)\omega^{(u+v+w)k}$	$2r' + \delta$	$-\delta\omega^{uk}$	$-r\omega^{uk}$	$-r'\omega^k$	$-r'\omega^{2k}$
$C_3^{(k,l)}$	1	0	0	ω^{uk}	0	$\delta\omega^{(u+v+w)k}$	$q\delta_{lu} - r'$	$-\delta\omega^{uk}$	$\delta\omega^{uk}$	$(q\delta_{lu} - r')\omega^k$	$(q\delta_{lu} - r')\omega^{2k}$
$C_4^{(k)}$	1	δs	q	$\delta s\epsilon^{uk} + \epsilon^{-2uk}$	$s\epsilon^{uk} + q\epsilon^{-2uk}$	$s\sum_{(u,v,w)}\epsilon^{k(u+v-2w)}$	s	$r\epsilon^{uk}$	0	0	0
$C_5^{(k)}$	1	1	0	$\epsilon^{uk} + \epsilon^{-2uk}$	$\delta\epsilon^{uk}$	$\delta\sum_{(u,v,w)}\epsilon^{k(u+v-2w)}$	δ	$-\delta\epsilon^{uk}$	0	0	0
$C_6^{(k,l,m)}$	1	2	δ	$\epsilon^{uk} + \epsilon^{ul} + \epsilon^{um}$	$\delta(\epsilon^{uk} + \epsilon^{ul} + \epsilon^{um})$	$\delta\sum_{[u,v,w]}\epsilon^{uk+vl+wm}$	$\delta(\omega^{k-l} + \omega^{l-k})$	0	0	0	0
$C_7^{(k)}$	1	0	$-\delta$	ϵ^{uk}	$-\delta\epsilon^{uk}$	0	0	$-\delta(\eta^{uk} + \eta^{\delta ukq})$	0	0	0
$C_8^{(k)}$	1	-1	δ	0	0	0	0	0	A_{uk}	$\delta\omega^k$	$\delta\omega^k$

$A_{uk} = \delta(\gamma^{uk} + \gamma^{ukq\delta} + \gamma^{ukq^2})$

are then counted by simple combinatorial methods. The classes of GL which split in SL have already been determined in Dickson [**1**] and so pose no problem. (It can be shown that the same splitting of classes takes place for the unitary case.) We restrict the characters of GL down to SL by using only the character values for those classes of GL which occur in SL; we do this by imposing the conditions set on k, l, m onto the abstract entries in the character table for GL. The range of the character parameters u, v, w must also be changed to reflect the fact that many parameter values result in the same character when restricted to SL.

The major difficulty lies in determining which restricted characters are reducible and how they split into irreducible components. If we let $d = \text{g.c.d.}(n, q-1)$ we see that the structure of SL is different for the various values which d can take on. If $d = 1$, then SL $\cong$ PSL and GL $=$ SL $\times Z(\text{GL})$ and since $Z(\text{GL})$ is abelian, every character of GL restricted to SL is irreducible. We show in § 4 that when $d = 3$, all the reducible restricted characters will split into 3 conjugate, irreducible components which are identical on all conjugacy classes of SL which are complete conjugacy classes of GL.

Definition 2.1. If χ and ψ are characters of group G, then their inner product, denoted by (χ, ψ) is given by the following:

$$(\chi, \psi) = \sum_{g_i} \frac{\chi(g_i)\overline{\psi(g_i)}}{|N(g_i)|}$$

where $g_i \in i$th conjugacy class and $N(g_i)$ is the centralizer of g_i. A basic result is that $(\chi, \chi) = 1$ if and only if χ is an irreducible character. Let χ be an irreducible character of GL and denote its restriction to SL by $\chi|\text{SL}$. The calculation $(\chi|\text{SL}, \chi|\text{SL})$ is in many cases difficult because one must work with polynomials in q and various summations of roots of unity. However, the results obtained in § 4 show that $(\chi|\text{SL}, \chi|\text{SL}) = 1$ or 3 and so, we can perform this calculation in a less precise fashion as we need only test to see if $(\chi|\text{SL}, \chi|\text{SL}) > 1$. This simplifies the calculations considerably in many cases.

The few remaining unknown entries in the character table for SL can be filled in using the orthogonality relations for characters and Gaussian sums.

3. Procedure for obtaining the characters of PSL$(3, q)$, $d = 3$**.** Once the table for SL is developed we can quickly obtain the characters of PSL by using the following theorem due to Frobenius.

THEOREM 3.1. *If $H \lhd G$ then every irreducible character of G which is constant on all classes of G equivalent modulo H is also an irreducible character of G/H. All irreducible characters of G/H are such characters of G.*

Since PSL $=$ SL$/Z$(SL) and Z(SL) $= \{$all scalar matrices of SL$\}$ we need only determine which classes of SL fuse in PSL under multiplication by scalar matrices with determinant 1. We then select those characters of SL which are constant on all classes which fuse in PSL.

4. A modification of Clifford's theorem. In this section we prove a stronger version of Clifford's theorem which is applicable only to the groups $\mathrm{GL}(n, q)$ and $\mathrm{U}(n, q^2)$ but which gives us more information as to how reducible restricted characters split into irreducible characters of $\mathrm{SL}(n, q)$, $\mathrm{SU}(n, q^2)$ respectively. We first require some preliminary information.

Definition 4.1. If χ is a character of G and $\sigma \in \mathrm{Aut}(G)$, then χ^σ is defined by:

$$\chi^\sigma(g) = \chi(g^\sigma).$$

We call χ, χ^σ conjugate characters of G. It can be shown that if χ is irreducible then χ^σ is also.

Definition 4.2. If $H \lhd G$ and ψ is an irreducible character of H, then we call ψ, ψ^g, $g \in G$ *conjugate characters of H relative to G.*

If ψ, ψ^g are conjugate relative to G, then they are equal on all classes of H which are complete classes of G.

Definition 4.3. If $H \leqq G$, then two characters χ, χ' of G are said to be *associates* if

$$\chi|H, \chi'|H \text{ have an irreducible component in common.}$$

We now state a theorem due to Clifford.

THEOREM 4.4. *If $H \lhd G$, G/H is cyclic, and χ is an irreducible character of G then $\chi|H = \psi_1 + \psi_2 + \ldots + \psi_t$ where $\{\psi_i\}$ is a complete set of irreducible characters of H conjugate relative to G. The character χ has $[G : H]/t$ associates in G and $t|[G : H]$.*

Since $\mathrm{SL} \lhd \mathrm{GL}$ and GL/SL is cyclic this theorem gives us nearly all we need to know in order to split a reducible restricted character of GL. In our case, where q is arbitrary, $t|[\mathrm{GL} : \mathrm{SL}] = (q - 1)$ is not specific enough. The following development refines this restriction on t.

LEMMA 4.5. *If $G = HK$ is a central product of group H with abelian group K and χ is an irreducible character of G, then $\chi|H$ is an irreducible character of H.*

Proof. If ψ, θ are irreducible characters of H, K respectively, then the character χ defined by $\chi(hk) = \psi(h)\theta(k)$ is an irreducible character of G; conversely, every irreducible character of G is equivalent to such a product of characters of H and K (see Gorenstein [**3**]). Thus $\chi(h) = \chi(h \cdot 1) = \psi(h) \cdot \theta(1) = \psi(h) \cdot 1 = \psi(h)$ since all the characters of an abelian group have degree 1. Since by the above $\chi|H = \psi$ we see that $\chi|H$ is an irreducible character of H.

Definition 4.6. Let

$$M(d) = \{A \in \mathrm{GL}(n, q) | \det A = \rho^{dk}, k = 1, \ldots (q - 1)/d\}$$

where $d = (n, q - 1)$ and ρ is a primitive element of $\mathrm{GF}(q)$.

LEMMA 4.6. (i) $M(d) \lhd \mathrm{GL}(n, q)$.

(ii) $\mathrm{GL}/M(d) \cong \sigma(d)$, *a cyclic group of order d.*

(iii) $M(d) = \mathrm{SL} \cdot Z(M(d))$ *is a central product with an abelian factor.*

Proof. The verification is quite direct.

These results can now be put together to get:

THEOREM 4.7. *With t defined in Clifford's theorem for $G = \mathrm{GL}(n, q)$ and $H = \mathrm{SL}(n, q)$, then $t|d$.*

Proof. Let χ be an irreducible character of GL. By Lemma 4.6 (i), (ii) and Clifford's theorem, $\chi|M(d)$ is either irreducible or it splits into t components where $t|[\mathrm{GL} : M(d)] = d$. But by Lemma 4.6 (iii) and Lemma 4.5, $(\chi|M(d))|\mathrm{SL}$ is irreducible. Since $\chi|\mathrm{SL} = (\chi|M(d))|\mathrm{SL}$ we see that $\chi|\mathrm{SL}$ is either irreducible or splits into t components where $t|d$.

From Theorem 4.7 we get the immediate useful result:

COROLLARY 4.8. *If χ is an irreducible character of* $\mathrm{GL}(3, q)$ *then*

$$(\chi|\mathrm{SL}, \chi|\mathrm{SL}) = 1 \text{ or } 3.$$

Identical theorems can be obtained for the unitary case by a few simple changes in the proofs and the definition of $M(d)$.

5. Ennola's conjecture. In [**2**] Ennola noted that the abstract character table for $U(n, q^2)$ could be obtained from the table for $\mathrm{GL}(n, q)$ for $n = 2, 3$ by the simple device of changing q to $-q$ throughout the table. He conjectured that this transformation would hold for all n. It is now reasonable to extend this conjecture to the groups SL, PSL and their unitary counterparts. The character tables for $\mathrm{SL}(3, q)$ and $\mathrm{SU}(3, q^2)$, $\mathrm{PSL}(3, q)$ and $\mathrm{PSU}(3, q^2)$ were found independently of each other. A comparison shows that Ennola's conjecture holds for these groups. In the case $n = 2$, $\mathrm{SL}(2, q) \cong \mathrm{SU}(2, q^2)$ and $\mathrm{PSL}(2, q) \cong \mathrm{PSU}(2, q^2)$ and the transformation $q \rightarrow -q$ did not alter the tables so Ennola's conjecture is valid for this case also. In § 7 we use Ennola's observation to combine the character tables for the linear and unitary cases to achieve greater economy of space.

6. Problems connected with extending the procedure for $n > 3$. To find $\mathrm{SL}(n, q)$ for some $n > 3$, the character table for $\mathrm{GL}(n, q)$ would have to be generated as described in Green's important paper [**5**]. When this first step is completed the major difficulty becomes evident, namely, the complexity of the character table for $\mathrm{GL}(n, q)$ for any $n > 3$. After restricting the characters of GL down to SL the next problem is determining which characters are reducible. For the cases $n = 2, 3$ this was done by taking the inner products $(\chi|\mathrm{SL}, \chi|\mathrm{SL})$. This is a very arduous task because the character entries are polynomials in q and various sums of roots of unity. Due to the complexity of the characters for $n > 3$, the prospect of carrying out such calculations is discouraging. It seems that a more sophisticated method must be found to determine which restricted characters are reducible.

After the reducible characters have been identified, they can then be split

TABLE 2.
Character Table for PSL$(3, q)$, PSU$(3, q^2)$ $d = 1, 3$

Conj. class	Canonical representation	Parameters	No. of classes	Centr. order				$1 \leqq u \leqq r' - 1$		$0 \leqq u \leqq d - d' - 1$	$1 \leqq u < v \leqq r'$ $v < w \leqq r$ $u + v + w = 0$ $(\mathrm{mod}\ r)$	$1 \leqq u \leqq r's$ $u \not\equiv 0\ (\mathrm{mod}\ s)$ $(u) = (u\delta q)$	$1 \leqq u \leqq t' - 1$ $(u) = (u\delta q) = (uq^2)$
					1	1	1	$r' - 1$	$r' - 1$	$d - d'$	$t'' - r''$	$3t'' - r'' - d'$	$2t''$
					χ_1	χ_{qs}	χ_{q^3}	$\chi_t^{(u)}$	$\chi_{qt}^{(u)}$	$\chi_{st'}$	$\chi_{st}^{(u,v,w)}$	$\chi_{rt}^{(u)}$	$\chi_{r^2s}^{(u)}$
C_1	$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$		1	$q^3r'rst$	1	qs	q^3	t	qt	st'	st	rt	r^2s
C_2	$\begin{bmatrix} 1 & & \\ 1 & 1 & \\ & & 1 \end{bmatrix}$		1	q^3r'	1	δq	δq	δs	q	$2r' + \delta$	$q + s$	$-\delta$	$-r$
$C_3^{(l)}$	$\begin{bmatrix} 1 & & \\ \theta^l & 1 & \\ & \theta^l & 1 \end{bmatrix}$	$0 \leqq l \leqq d - 1$	d	q^2	1	0	0	1	0	$q\delta_{uk} - r'$	δ	$-\delta$	δ
$C_4^{(k)}$	$\begin{bmatrix} \rho^k & & \\ & \rho^k & \\ & & \rho^{-2k} \end{bmatrix}$	$1 \leqq k \leqq r' - 1$	$r' - 1$	$qr'rs$	1	δs	q	$\delta s\epsilon^{3uk} + \epsilon^{-6uk}$	$s\epsilon^{3uk} + q\epsilon^{-6uk}$	s	$s(\epsilon^{-3uk}$ $+\epsilon^{-3vk} + \epsilon^{-3wk})$	$r\epsilon^{3uk}$	0
$C_5^{(k)}$	$\begin{bmatrix} \rho^k & & \\ 1 & \rho^k & \\ & & \rho^{-2k} \end{bmatrix}$	$1 \leqq k \leqq r' - 1$	$r' - 1$	qr'	1	1	0	$\epsilon^{3uk} + \epsilon^{-6uk}$	$\delta\epsilon^{3uk}$	δ	$\delta(\epsilon^{-3uk}$ $+\epsilon^{-3vk} + \epsilon^{-3wk})$	$-\delta\epsilon^{3uk}$	0
C_6'	$\begin{bmatrix} 1 & & \\ & \omega & \\ & & \omega^2 \end{bmatrix}$		$1 - d'$	r^2	1	2	δ	3	3δ	$\delta(\omega^{k-l} + \omega^{l-k})$	$3\delta(\omega^{u-v} + \omega^{v-u})$	0	0
$C_6^{(k,l,m)}$	$\begin{bmatrix} \rho^k & & \\ & \rho^l & \\ & & \rho^m \end{bmatrix}$	$1 \leqq k < l \leqq r'$ $l < m \leqq r$ $k + l + m = 0\ (\mathrm{mod}\ r)$	$t'' - r''$	$r'r$	1	2	δ	$\epsilon^{3uk} + \epsilon^{3ul} + \epsilon^{3um}$	$\delta(\epsilon^{3uk} + \epsilon^{3ul} + \epsilon^{3um})$	$\delta(\omega^{k-l} + \omega^{l-k})$	$\delta \sum_{[u,v,w]} \epsilon^{uk+lv+mw}$	0	0
$C_7^{(k)}$	$\begin{bmatrix} \rho^k & & \\ & \sigma^{-\delta k} & \\ & & \sigma^{-qk} \end{bmatrix}$	$1 \leqq k \leqq r's$ $k \not\equiv 0\ (\mathrm{mod}\ s)$ $C^{(k)} = C^{(\delta qk)}$	$3t'' - r'' - d'$	$r's$	1	0	$-\delta$	ϵ^{3uk}	$-\delta\epsilon^{3uk}$	0	0	$-\delta(\eta^{3uk} + \eta^{3\delta quk})$	0
$C_8^{(k)}$	$\begin{bmatrix} \tau^k & & \\ & \tau^{\delta kq} & \\ & & \tau^{kq^2} \end{bmatrix}$	$1 \leqq k \leqq t' - 1$ $C^{(k)} = C^{(\delta qk)} = C^{(q^2k)}$	$2t''$	t'	1	-1	δ	0	0	0	0	0	B_{uk}

$$B_{uk} = \delta(\gamma_2^{uk} + \gamma_2^{\delta quk} + \gamma_2^{q^2uk})$$

up into irreducible components using the results described in § 4. However, this splitting process poses a difficulty on all those classes of SL which are not complete conjugacy classes in GL. To determine how the characters split over these classes we must use the various orthogonality relations and other known properties of character tables. For $n = 3$ there were only three 3×3 blocks of entries to be calculated in this manner, but for larger values of n the blocks of undetermined entries will become both more numerous and larger and filling them in will be a much more difficult task.

Finally, the size and complexity of the resulting character table for SL(n, q) for even $n = 4$, would prohibit its publication and probably discourage its use.

As stated above, the next problem is to find some means of easily determining which characters of GL(n, q) are reducible when restricted to SL(n, q). Since the sets of conjugacy classes of GL with the same canonical form and the sets of characters having the same degree are linked by a 1-1 correspondence, we might at first suspect that the characters and conjugacy classes of GL which split in SL would be related by the 1-1 correspondence, but such is not the case. In fact, we find, in dealing with GL$(3, q)$, that only one type of conjugacy class splits in SL$(3, q)$ whereas, two types of characters split. Thus there does not seem to be any readily apparent relationship between classes and characters which split when restricted to SL. Some success has been realized by examining the number of characters of the same degree. This number must be divisible by $(q - 1)$ in order for all the characters to be irreducible upon restriction to SL. Furthermore, the reducible characters occur in sets of $(q - 1)/t$. Thus the type and number of many reducible characters can be determined.

7. Character tables for, SL$(3, q)$**,** SU$(3, q^2)$**,** PSL$(3, q)$**,** PSU$(3, q^2)$**,** q **arbitrary.** The following notation is used for the tables:

$$\delta = \begin{cases} +1 & \text{for SL}(3, q),\ \text{PSL}(3, q) \\ -1 & \text{for SU}(3, q^2),\ \text{PSU}(3, q^2) \end{cases}$$

$$\rho^r = 1,\ \sigma^s = \rho,\ \tau^t = 1,\ \theta^3 \neq 1,\ \omega^3 = 1,\ \theta \in \begin{cases} \mathrm{GF}(q) & \text{if } \delta = 1 \\ \mathrm{GF}(q^2) & \text{if } \delta = -1 \end{cases}$$

$$r = (q - \delta) \qquad r' = r/d \qquad r'' = r'(1 + \delta)/2$$

$$s = (q + \delta) \qquad s' = s/d \qquad t'' = (t' - 1)/6$$

$$t = q^2 + \delta q + 1 \qquad t' = t/d$$

$$d = (n, r) = 1 \text{ or } 3 \qquad d' = (3 - d)/2$$

q = a prime power

$$\epsilon^r = 1 \qquad \eta^s = \epsilon = \gamma_1{}^t \qquad \gamma = \gamma_1{}^r \qquad \gamma_2 = \gamma_1{}^{r'},$$

$\sum_{(x,y,z)}$ means a sum over the cyclic permutations of x, y, z,
$\sum_{[x,y,z]}$ means a sum over all the permutations of x, y, z.

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