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CORRIGENDUM

An introduction to Bayesian inference in gravitational-wave astronomy: parameter estimation, model selection, and hierarchical models—Corrigendum

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We correct two errors in Thrane and Talbot (2019).

1. In the original version of this article, we included a subsection in Appendix E, "Selection effects with a single event." This section included formulas with errors including Eq. 89 and Eq. 95 of the arxiv version (Eq. E2 and Eq. E8 in the version published in PASA). Moreover, the section included a conceptual error since the idea of selection effects for single events does not make sense. Selection effects are intrinsically related to population studies, so they simply do not affect the analysis of single detections. It is interesting to consider how this comes about mathematically. While the single-event det likelihood gains a factor of p_{det}^{-1} (as correctly noted in the original article), the single-event det prior picks up a compensating factor of p_{det}, because the prior for detected events is not the same as the original (no det) prior. Since the det posterior is proportional to the product of the likelihood and the prior, these two factors cancel, giving the original (no det) likelihood. A revised version of the appendix is presented below.
2. In eight places we referred to "the odds ratio." However, we should have referred simply to "the odds." In statistics, the odds refers to a ratio of probabilities. When we multiply the Bayes factor by the prior odds, we obtain the posterior odds. The odds ratio, which is also a statistical term, refers to a ratio of ratios.

The following is the revised version of Appendix E.

Appendix E. Selection Effects

In this section, we discuss how to carry out inference while taking into account selection effects, which arise from the fact that some events are easier to detect than others. We loosely follow the arguments from Abbott et al. (2016); however, see also Mandel et al. (2018); Fishbach et al. (2018).

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Some gravitational-wave events are easier to detect than others. All else equal, it is easier to detect binaries if they are closer, higher mass (at least, up until the point that they start to go out of the observing band), and with face-on/off inclination angles. More subtle selection effects arise due to black hole spin (see, e.g., Ng et al., 2018). Typically, a gravitational-wave event is said to have been detected if it is observed with a matched-filter signal-to-noise ratio—maximized over extrinsic parameters $\theta_{\text{extrinsic}}$—above some threshold ρ_{th}

$$\rho'_{\text{mf}} \equiv \max_{\theta_{\text{extrinsic}}} (\rho_{\text{mf}}) > \rho_{\text{th}}. \tag{1}$$

Usually, $\rho_{\text{th}} = 8$ for a single detector or $\rho_{\text{th}} = 12$ for a ≥ 2 detector network.

Selection effects are characterised by p_{det}, the probability that a signal exceeds the detection threshold. There are different ways to calculate p_{det} in practice. The probability density function for ρ_{mf} given θ—the distribution of ρ_{mf} arising from random noise fluctuations—is a normal distribution with mean ρ_{opt} and unit variance

$$p(\rho'_{\text{mf}}|\theta) = \frac{1}{2\pi} \exp\left(-\frac{1}{2}\left(\rho'_{\text{mf}} - \rho_{\text{opt}}(\theta)\right)^2\right), \tag{2}$$

see Fig. 1. Thus,

$$p_{\text{det}}(\theta) = \int_{\rho_{\text{th}}}^{\infty} dx \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(x - \rho_{\text{opt}}(\theta)\right)^2\right) \tag{3}$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{\rho_{\text{th}} - \rho_{\text{opt}}(\theta)}{\sqrt{2}}\right). \tag{4}$$

Alternatively, one may express p_{det} as the ratio of the "visible volume" $\mathcal{V}(\theta)$ to the total spacetime volume $\mathcal{V}_{\text{tot}}$

$$p_{\text{det}}(\theta) = \frac{\mathcal{V}(\theta)}{\mathcal{V}_{\text{tot}}}. \tag{5}$$

The visible volume is typically calculated numerically with injected signals.

Given a population of N events,

$$\mathcal{L}(d, N|\Lambda, \text{det}) = \frac{1}{p_{\text{det}}(\Lambda|N)} \mathcal{L}(d, N|\Lambda, R). \tag{6}$$

In analogy to Eq. 5, the p_{det} normalization factor can be calculated using the visible volume as a function of the hyper-parameters Λ

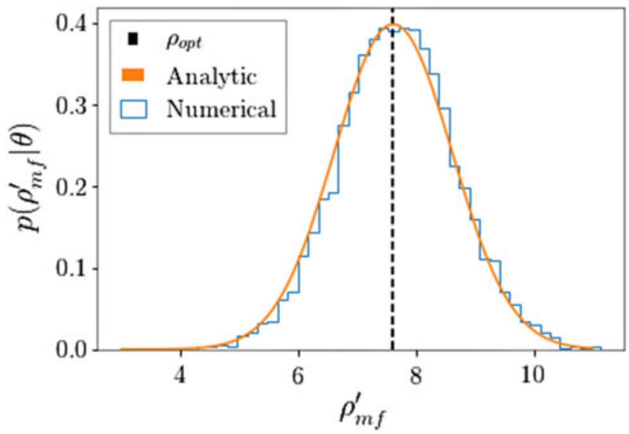

Figure 1. The distribution of matched filter signal-to-noise ratio maximized over phase for the same template in many noise realisations (blue). The distribution peaks at $\rho_{\rm opt} = 7.6$ (dashed black). The theoretical distribution (Eq. 2) is shown in orange.

$$\mathcal{V}(\Lambda) \equiv \int d\theta \mathcal{V}(\Lambda)\pi(\theta|\Lambda). \tag{7}$$

Naively, one might expect that

$$p_{\rm det}(\Lambda|N) = \left(\frac{\mathcal{V}(\Lambda)}{\mathcal{V}_{\rm tot}}\right)^N, \tag{8}$$

but this expression is incorrect because it does not marginalize over the Poisson-distributed rate, which ends up changing the answer. Marginalizing over the rate, we obtain

$$\begin{aligned} p_{\rm det}(\Lambda|N) &= \int dR \left(\frac{\mathcal{V}(\Lambda)}{\mathcal{V}_{\rm tot}}\right)^N \pi(N|R)\pi(R) \\ &= \int dR \left(\frac{\mathcal{V}(\Lambda)}{\mathcal{V}_{\rm tot}}\right)^N \left[e^{-R\mathcal{V}(\Lambda)} \frac{\mathcal{V}(\Lambda)^N R^N}{N!}\right] \pi(R) \\ &= \left(\frac{\mathcal{V}(\Lambda)}{\mathcal{V}_{\rm tot}}\right)^N \left[\int dR\, e^{-R\mathcal{V}(\Lambda)} \frac{\mathcal{V}(\Lambda)^N R^N}{N!}\right] \pi(R). \end{aligned} \tag{9}$$

Note that $p_{\rm det}$ depends on our prior for the rate R. If we choose a uniform-in-log prior $\pi(R) \propto 1/R$, we obtain

$$p_{\rm det}(\Lambda|N) \propto \left(\frac{\mathcal{V}(\Lambda)}{\mathcal{V}_{\rm tot}}\right)^N, \tag{10}$$

which reproduces the results from Abbott et al. (2018). Note that

$$\mathcal{L}(d|\Lambda, {\rm det}) \neq \int d\theta \mathcal{L}(d|\theta, {\rm det})\, \pi(\theta|\Lambda). \tag{11}$$

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