Black Holes from Stars to Galaxies – Across the Range of Masses
Proceedings IAU Symposium No. 238, 2006
V. Karas & G. Matt, eds.

doi:10.1017/S1743921307005789

Brownian motion of black holes in stellar systems with non-Maxwellian distribution of the field stars

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Abstract. A massive black hole at the center of a dense stellar system, such as a globular cluster or a galactic nucleus, is subject to a random walk due gravitational encounters with nearby stars. It behaves as a Brownian particle, since it is much more massive than the surrounding stars and moves much more slowly than they do. If the distribution function for the stellar velocities is Maxwellian, there is a exact equipartition of kinetic energy between the black hole and the stars in the stationary state. However, if the distribution function deviates from a Maxwellian form, the strict equipartition cannot be achieved.

The deviation from equipartition is quantified in this work by applying the Tsallis q-distribution for the stellar velocities in a q-isothermal stellar system and in a generalized King model.

Keywords. black holes, equipartition, Tsallis distribution, King models

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1. Introduction

A massive black hole as a Brownian particle conducts to a equipartition of kinetic energy at the equilibrium, when the distribution function of stellar velocities is Maxwellian. When it is non-Maxwellian is not surprising to find a deviation from equipartition. This deviation is defined as $\eta = (M\langle V^2\rangle)/(m\langle v^2\rangle)$, where M and V represent the mass and velocity of the black hole, and m and v masses and velocities of stars. Following Chatterjee *et al.* (2002) we have $\eta = [3\int_0^\infty f(r,v)vdv \ \int_0^\infty f(r,v)v^2dv]/[f(r,0) \ \int_0^\infty f(r,v)v^4dv]$, with $f(r,v)$ representing the distribution function for stars field. Evidently, for Maxwellian distribution $\eta = 1$. However, astrophysical systems are non extensive since they are subject to long range interactions. Boltzmann-Gibbs (BG) thermostatistics is able to predictions in extensive systems, in the sense that microscopic interactions are short or ignored, temporal or spatial memory effects are short range or does not exist and is valid ergodicity in the phase space. BG domain is enlarged by non extensive statistical mechanics, based on Tsallis entropy (Tsallis 1988) and in the q-distribution function: $p(x) \propto [1-(1-q)\beta x]^{1/(1-q)}$. Boltzmann distribution is recovered in the $q \to 1$ limit, and all the usual statistical mechanics as well. A summary about mathematical properties of these functions can be found in Umarov *et al.* (2006).

2. Extended stellar models, deviation from equipartition and results

The isothermal sphere is the simplest model for spherical systems. It was generalized by Lima & Souza (2005) based on the distribution function $f_q(v) \propto \left[1-(1-q)v^2/(2\sigma^2)\right]^{\frac{1}{1-q}}$ and the deviation results in $\eta(q) = (7-5q)/[2(2-q)]$. Most commonly used are King models, based on truncated isothermal spheres. Extended King models were presented in

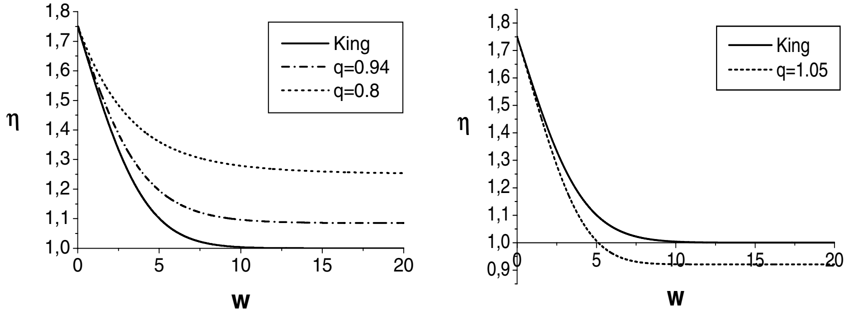

Figure 1. At the left, deviation for q-King model with $q = 0.94$, and $q = 0.8$. The limit value is $\eta = (7 - 5q)/2(2 - q)$. At the right, η for $q = 1.05$. King means $q = 1$, the usual King model. Strict equipartition implies $\eta = 1$.

Fa & Pedron (2001) and applied to fit surface brightness of the NGC3379 and 47TUC with excellent results. The distribution function in q-King models is $f_{k_q}(\epsilon) = \rho_1(2\pi\sigma^2)^{-3/2}\{[1+(1-q)\epsilon/\sigma^2]^{1/(1-q)} - 1\}$ for $\epsilon > 0$ and $f_{k_q}(\epsilon) = 0$ for $\epsilon \leqslant 0$. Here ϵ is the relative energy $\epsilon = \psi - v^2/2 > 0$ and $\psi = -\phi(r) + \phi_0$ the relative potential. The central potential is $W = \psi(0)/\sigma^2$ and in the $\psi(0)/\sigma^2 \to \infty$ limit the isothermal sphere is recovered. For such model the η value for $q < 1$ and $w = -\phi(r)$ is

$$\eta(q, w) = \frac{3}{2}\frac{\left[a^{-1}A^{\frac{2-q}{a}}\beta(y, 1, \frac{2-q}{1-q}) - w\right]\left[a^{\frac{-3}{2}}A^{\frac{5-3q}{2a}}\beta(y, \frac{3}{2}, \frac{2-q}{1-q}) - \frac{2}{3}w^{3/2}\right]}{(A^{1/a} - 1)\left[a^{-5/2}A^{(7-5q)/2a}\beta(y, \frac{5}{2}, \frac{2-q}{1-q}) - \frac{2}{5}w^{5/2}\right]} \tag{2.1}$$

where β is the incomplete Beta function, $a = 1 - q$, $y = \frac{w}{1/a+w}$, and $A = 1 + aw$. For $q > 1$ we obtain

$$\eta(q, w) = \frac{3}{2}\frac{\left[b^{-1}\tilde{A}^{(2-q)/b}I_1 - w\right]\left[b^{-\frac{3}{2}}\tilde{A}^{(5-3q)/2b}I_2 - \frac{2}{3}w^{3/2}\right]}{(\tilde{A}^{-1/b} - 1)\left[b^{-\frac{5}{2}}\ \tilde{A}^{(7-5q)/2b}I_3 - \frac{2}{5}w^{5/2}\right]} \tag{2.2}$$

where $b = q - 1$, $\tilde{A} = 1 - bw$, and $xm = \frac{w}{1/b-w}$. Furthermore, $I_1 = \int_0^{xm}(1+x)^{-1/b}dx$, $I_2 = \int_0^{xm}(1+x)^{-1/b}x^{1/2}dx$, and $I_3 = \int_0^{xm}(1+x)^{-1/b}x^{3/2}dx$ with the conditions $\tilde{A} = 1 - bw \geqslant 0$ and $q \leqslant 1 + \frac{1}{w}$.

In the Figure 1 results indicate that there is a deviation from equipartition *a priori*, even at very long time-scales. The equipartition is never achieved in both cases ($q < 1$ and $q > 1$). The q parameter will be, in some way, dictated by the system.

References

Chatterjee, P., Hernquist, L. & Loeb, A. 2002, Phys. Rev. Lett., 88, 121103
Fa, K. S. & Pedron, I. T. 2001, astro-ph/0108370
Lima, J. A. S. & Souza, R. E. 2005, Physica A, 350, 303
Tsallis, C. 1988, J. Stat. Phys., 52, 479
Umarov, S., Tsallis, C., Gell-Mann, M. & Steinberg, S. 2006, cond-mat/0606038

https://doi.org/10.1017/S1743921307005789 Published online by Cambridge University Press