GRAVITATIONAL AND DYNAMICAL INSTABILITIES OF A DECELERATING PLANE-PARALLEL SLAB OF FINITE THICKNESS

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Abstract: In order to explore how supernova blast waves might catalyze star formation, we investigate the stability of a slab of decelerating gas of finite thickness. We examine the early work in the field by Elmegreen and Lada and Elmegreen and Elmegreen and demonstrate that it is flawed. Contrary to their claims, blast waves can indeed accelerate the rate of star formation in the interstellar medium. Also, we demonstrate that in an incompressible fluid, the symmetric and antisymmetric modes in the case of zero acceleration transform continuously into Rayleigh-Taylor and gravity-wave modes as acceleration grows more important.

Shock fronts in the interstellar medium may be generated by several common mechanisms: supernovae, strong stellar winds, ionization fronts, to name a few. As these shocks sweep through the interstellar medium they compress the ambient gas into shells of considerably higher density. In increasing the density of the gas, the shocks decrease the characteristic gravitational collapse time, $t_{ff} \sim \sqrt{1/G\rho}$. However, in these compressed shells, Jeans-collapse theory is no longer valid because the shell thicknesses of interest are much smaller than the Jeans lengths of the compressed gas. There has been considerable debate in the literature regarding whether or not shocks can accelerate the process of star formation by enabling gravitational collapse to proceed more quickly. We intend in this paper to illuminate some of the issues in that debate by demonstrating that certain claims in the literature are conceptually ill-grounded, and by presenting a toy model of our own to illustrate the relevant physical phenomena.

We concern ourselves in this paper primarily with the instabilities of a plane-parallel slab of finite thickness, bounded by contact discontinuities on both sides, where the pressures on the two sides are, in general, dissimilar. Elmegreen and Lada (1977) examined the critical stability of an isothermal, self-gravitating, decelerating slab of gas bounded by contact discontinuities on both surfaces. They found that no instabilities arise in the slab until its age is of order t_{ff} in the unshocked medium (t_{norm}), i.e. shocks do not accelerate star formation. Elmegreen and Elmegreen (1978) calculated dispersion relations for the modes of a slab of isothermal, self-gravitating, stationary gas bounded by contact discontinuities on both surfaces. They found that the slab becomes unstable to deformational instabilities on timescales much shorter than t_{norm}, but that instabilities to gravitational collapse do not arise until the age of the slab is of order t_{norm}, in agreement with Elmegreen and Lada (1977). Vishniac (1983) found dispersion relations for the gravitationally unstable modes of an infinitesimally thin shell of gas having a polytropic equation of state and bounded on one side by shock jump conditions and on the other by a contact discontinuity. His results show that gravitational collapse of the shell can

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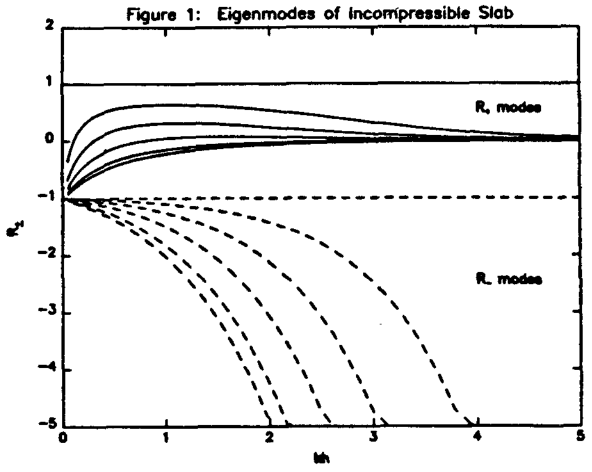

Figure 1 illustrates how the eigenmodes of an incompressible slab change with changing $\mathcal{P}$ and differ at differing dimensionless wavenumber kh. The quantity $R \equiv -\eta/\zeta$, where η and ζ are the amplitudes of the perturbative displacements of the top and bottom surfaces, characterizes a given mode. The solid lines give the R_+ modes for $\beta = 2$ $(p_2 = 2p_1)$ and $\mathcal{P} = 0.0, 0.2, 0.5, 1.0, 2.0, 3.0$; the dashed lines give the R_- modes for the same values of $\mathcal{P}$. The quantities $R_\pm$ decrease monotonically with increasing $\mathcal{P}$. Note that for $\mathcal{P} = 0$, the R_+ modes is symmetric $(R_+ = 1)$, and the R_- mode is antisymmetric $(R_- = -1)$. When $\mathcal{P} \neq 0$, at large kh (small wavelength), $R_+ \rightarrow 0$, and $R_- \rightarrow -\infty$, that is, the surfaces decouple into separate waves on top and bottom. As $\mathcal{P}$ grows, decoupling occurs at longer and longer wavelengths (smaller kh).

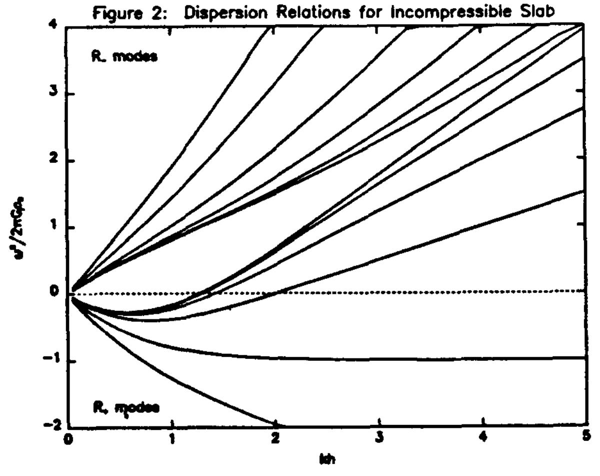

Figure 2 displays the dispersion relations of R_+ and R_- modes - dimensionless frequency squared $(\omega^2/2\pi G\rho_0)$ versus dimensionless wavenumber (kh). The relations are plotted for $\beta = 2$ and $\mathcal{P} = 0.0, 0.2, 0.5, 1.0, 2.0, 3.0$. For R_- modes, $\omega^2 > 0$ always, the modes are always stable, and ω^2 always increases with increasing $\mathcal{P}$. The R_+ modes go unstable $(\omega^2 < 0)$ for at least some kh, no matter what the value of $\mathcal{P}$, and ω^2 decreases with increasing $\mathcal{P}$. As $\mathcal{P}$ becomes large, the R_+ modes become Rayleigh-Taylor modes, and the R_- modes become gravity waves. That this physics should emerge is not surprising.

occur at times considerably smaller than t_{norm} in the unshocked medium – that shocks can accelerate the rate of star formation in the ISM.

Elmegreen and Lada (1977) begin with an isothermal, plane-parallel slab bounded by two contact discontinuities at pressures p_1 and p_2. The $\hat{z}$ direction points perpendicular to the slab and the $\hat{x}$ direction along it. They then introduce a density perturbation

$$\rho_1 = \rho_0 e^{ikx}\theta(z).$$

To this system they apply boundary conditions

$$\theta(z_1) = \theta(z_2) = 0,$$

where $z_1 \equiv z(p_1)$ and $z_2 \equiv z(p_2)$ are defined in the unperturbed system. In an isothermal slab the pressure perturbation, P_1, obeys $P_1 = (const.)\rho_1$, so

$$P_1(z_1) = P_1(z_2) = 0,$$

and

$$P(z_1) = P_0(z_1) = p_1$$
$$P(z_2) = P_0(z_2) = p_2.$$

If the boundary of the perturbed slab is displaced by some $\eta(x,t) \ll |z_1 - z_2|$, then

$$P(z_1 + \eta) = P(z_1) = p_1.$$

In this system, dP/dz is monotonic and nonzero at the boundary; therefore, $\eta = 0$ by necessity, and the boundaries must be rigid. Elmegreen and Lada find an instability criterion valid only for a small and nonphysical subset of the possible modes – those for which the boundaries of the slab remain fixed.

Elmegreen and Elmegreen (1978) consider an isothermal, plane-parallel slab of gas bounded by two contact discontinuities at the same pressure; thus, the slab is symmetric about its midplane. They assume all perturbed quantities take the form $\hat{f}(x,z,t) = f(z)e^{i(kx+\omega t)}$. Allowing the top and bottom surfaces to ripple either symmetrically or antisymmetrically about the midplane according to the displacement $\hat{\eta}(x,t) = \eta e^{i(kx+\omega t)}$, they calculate numerically the dispersion relations, $\omega(k)$, for the symmetric and antisymmetric modes. They find that the antisymmetric modes are always stable and that the symmetric modes are unstable to wavelengths longer than of order the thickness of the slab. However, the symmetric modes merely deform the slab, causing it to quilt – they do not collapse gravitationally. Elmegreen and Elmegreen then solve for the time when gravitational instability does set in by calculating when the gas contained within one characteristic wavelength of the symmetric mode will collapse. They fail to recognize that collapse over much longer wavelengths will occur much earlier.

In order to explore the stability of a decelerating plane-parallel slab of gas bounded by two contact discontinuities at differing pressures, p_1 and $p_2 = \beta p_1$, we consider the case of the incompressible slab, which we can solve analytically. Assuming perturbed quantities take the form $\hat{f}(x,z,t) = f(z)e^{i(kx+\omega t)}$, we allow the displacements of the two surfaces to ripple according to $\hat{\eta}(x,t) = \eta e^{i(kx+\omega t)}$ and $\hat{\zeta}(x,t) = \zeta e^{i(kx+\omega t)}$. Since we introduce two degrees of freedom, namely η and ζ, into the problem, we obtain the two dispersion relations

$$\omega_{\pm}^2 = 2\pi G\rho_0 \left[(R_{\pm}e^{-kh} - 1) + \frac{kh}{e^{kh} - e^{-kh}} \left\{ \left(e^{kh} - 2R_{\pm} + e^{-kh}\right) - \frac{(\beta - 1)}{2}\mathcal{P}\left(e^{kh} + 2R_{\pm} + e^{-kh}\right) \right\} \right]$$

$$R_{\pm} = \frac{U(e^{kh} + e^{-kh}) \mp \sqrt{U^2(e^{kh} - e^{-kh})^2 + 4V^2}}{2(V - U)}$$

$$U = kh(\beta - 1)\mathcal{P}$$

$$V = 1 - e^{-2kh} - 2kh,$$

where ρ_0 is the density of the slab, h is the slab thickness, σ is the surface density of the slab, $R \equiv -\eta/\zeta$ describes the eigenmodes, and $\mathcal{P} \equiv p_1/\pi G\sigma^2$ is a dimensionless quantity which describes the relative importance of self-gravity in determining the structure of the slab. We shall refer to the two eigenmodes as $R_{\pm}$ modes. We plot $R_{\pm}$ in Fig. 1 and $\omega_{\pm}^2$ in Fig. 2. When $\beta = 1$, our dispersion relations are qualitatively identical to those that Elmegreen and Elmegreen find for an isothermal slab bounded by equal pressures. We believe this correspondence justifies our admittedly nonphysical assumption of incompressibility.

In summary, we have shown that Elmegreen and Lada (1977) and Elmegreen and Elmegreen (1978) made conceptual errors in their treatments of the instabilities of isothermal slabs of gas. They concluded incorrectly that shocks do not accelerate star formation. A correct approach to the shock induced gravitational instability, albeit in the thin shell approximation, is given by Vishniac (1983). We have also solved for the stability of a slab of incompressible gas bounded by differing pressures. The dispersion relations we find demonstrate that the symmetric and antisymmetric modes of Elmegreen and Elmegreen (1978) are related to Rayleigh-Taylor and gravity waves, respectively.

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