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LYUAPONOV EXPONENTS FOR OPEN BILLIARD SYSTEMS

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In this thesis, we investigate the largest Lyapunov exponent for open billiards in both two- and higher-dimensional Euclidean spaces. In $\mathbb{R}^2$, we estimate the largest Lyapunov exponent λ_1 for open billiards, demonstrating its continuity and differentiability with respect to a small perturbation parameter α. Extending this study to $\mathbb{R}^n$ for $n \geq 3$, we prove similar results for the largest Lyapunov exponent for open billiards in higher dimensions. Additionally, we consider the billiard flow in the exterior of several (at least three) balls in $\mathbb{R}^3$ with centres lying on a plane. We assume that the balls satisfy the no-eclipse condition (**H**) and their radii are small compared with the distances between their centres. We prove that with respect to any Gibbs measure on the nonwandering set of the billiard map, the two positive Lyapunov exponents are different: $\lambda_1 > \lambda_2 > 0$. These findings enhance our understanding of chaotic dynamics and could be applied to similar physical systems, such as Lorentz gases.

Some of this research has appeared in [1–3].

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