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Drift, diffusion and divergence

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Turbulent Taylor–Couette flow displays traces of axisymmetric Taylor vortices even at high Reynolds numbers. With this motivation, Feldmann & Avila (2025) *J. Fluid Mech*, **1008**, R1, carry out long-time numerical simulations of axisymmetric high-Reynolds-number Taylor–Couette flow. They find that the Taylor vortices, using the only degree of freedom that remains available to them, carry out Brownian motion in the axial direction, with a diffusion constant that diverges as the number of rolls is reduced below a critical value.

Key words: Taylor–Couette flow, transition to turbulence

1. Introduction

In 1923, Taylor published his ground-breaking experiment and linear stability calculation, whose agreement demonstrated the validity of the Navier–Stokes equations. Since then, Taylor–Couette flow has served as one of the protypical systems in fluid dynamics. In the Taylor–Couette experiment, fluid is confined between two concentric cylinders which rotate at different angular velocities. In laminar Taylor–Couette flow, the motion is purely azimuthal and fluid particles at different radii do not mix. Increasing the angular velocity difference past a critical value leads to the formation of Taylor vortices, toroidal rolls in which circular motion in the meridional (r, z) plane redistributes fluid and angular momentum between the radii.

Ever since Taylor described and explained the onset of axisymmetric Taylor-vortex flow, an extravagant profusion of three-dimensional patterns of extraordinary variety, beauty and complexity have been discovered experimentally and numerically (e.g. Andereck *et al.* 1986; Weisshaar *et al.* 1991; Chossat & Iooss 1994; Altmeyer *et al.* 2012; Deguchi & Altmeyer 2013; Akinaga *et al.* 2018). The mathematics of what is called variously equivariant bifurcation theory, symmetry and pattern formation has been brought to bear to predict and explain these spirals and ribbons, twists and waves, modulation and bursts.

Turbulence in Taylor–Couette flow has also been studied, both at high Reynolds number and in the transitional range at low Reynolds number (e.g. Coles 1965; Goharzadeh & Mutabazi 2001; Prigent *et al.* 2002; Shi *et al.* 2013; Lemoult *et al.* 2016). But who would have thought that there was something new to be learned about turbulence from axisymmetric Taylor–Couette flow?

2. Summary of paper

It has long been known that the Taylor-vortex structure persists even far into the turbulent regime, i.e. that turbulence is superposed on Taylor vortices (e.g. Lathrop *et al.* 1992; Dong 2007; Huisman *et al.* 2014; Grossmann *et al.* 2016); long-time averaging accentuates the features of these ghostly vortices. Eckhardt *et al.* (2020) have argued that, under certain hypotheses, transport of angular momentum by chaotic fluctuations in axisymmetric Taylor–Couette flow reproduces the transport associated with the axisymmetric component of turbulent solutions to the full three-dimensional equations. This suggests that the axisymmetric problem could be viewed, not merely as a first step towards turbulence (laminar → axisymmetric Taylor-vortex flow → three-dimensional patterns → turbulence), but as a model for its mean (necessarily axisymmetric) properties. Feldmann & Avila (2025) have carried out long-time axisymmetric simulations of Taylor–Couette flow as a possible route towards studying turbulent structures.

Axisymmetric Taylor-vortex flow consists of an axial stack of toroidal vortices. The vortices are approximately circular, so that the number of vortices is close to the axial-length-to-radial-gap ratio $\varGamma$. Feldmann & Avila (2025) observe that the number of vortices remains constant over the course of a simulation. Such a one-dimensional periodic structure is highly constrained and so its possible dynamics are limited: the only remaining possible motion is an axial jiggle or drift of the entire stack of vortices. Feldmann & Avila (2025) find that, for a relatively long system, the rolls carry out diffusive drift (Brownian motion) so that the variance of the phase grows linearly in time. Moreover, the effective diffusion coefficient diverges following a power law as a threshold axial length (or number of rolls) $\varGamma_c$ is approached from above. For a shorter axial length, although there may be an immediate adjustment of the position, the rolls quickly becomes quasi-stationary, with only weak chaotic motion about a fixed location. For the parameters used by Feldmann & Avila (2025), $\varGamma_c = 10$; see figure 1. The significance of this sharp threshold is unknown.

Although this is an interesting puzzle by itself, its importance is increased by its generality. Many hydrodynamic systems are driven by an imposed gradient of some quantity. Rolls appear as a means of redistributing this quantity: azimuthal or streamwise velocity for Taylor–Couette, plane Couette or Poiseuille flow, temperature for Rayleigh–Bénard convection, concentration for a binary fluid. Drift has been observed in these other systems (Xi *et al.* 2006; Kreilos *et al.* 2014) and according to Feldmann & Avila (2025), the drift appears to be of the same type.

Exploiting the analogy between axisymmetric Taylor–Couette flow and two-dimensional Rayleigh–Bénard convection (Veronis 1970), Eckhardt *et al.* (2020) have proposed a mapping from the two Reynolds numbers (inner and outer, or equivalently, shear Re_S and rotation $R_\varOmega$ ($Re_S \equiv Ud/\nu$ and $R_\varOmega \equiv 2d\varOmega/U$ where d is the gap width between the outer and inner cylinders, $\varOmega$ is the angular velocity of the outer cylinder, and U is the difference between the angular velocities of the inner and outer cylinders times the inner cylinder radius.)) of Taylor–Couette flow (Dubrulle *et al.* 2005) to the single Rayleigh number Ra of Rayleigh–Bénard convection. Feldmann & Avila (2025) have provided support for this analogy by showing that the diffusion coefficient of the axial drift was the same for different parameter pairs $(Re_S, R_\varOmega)$ yielding the same value of Ra.

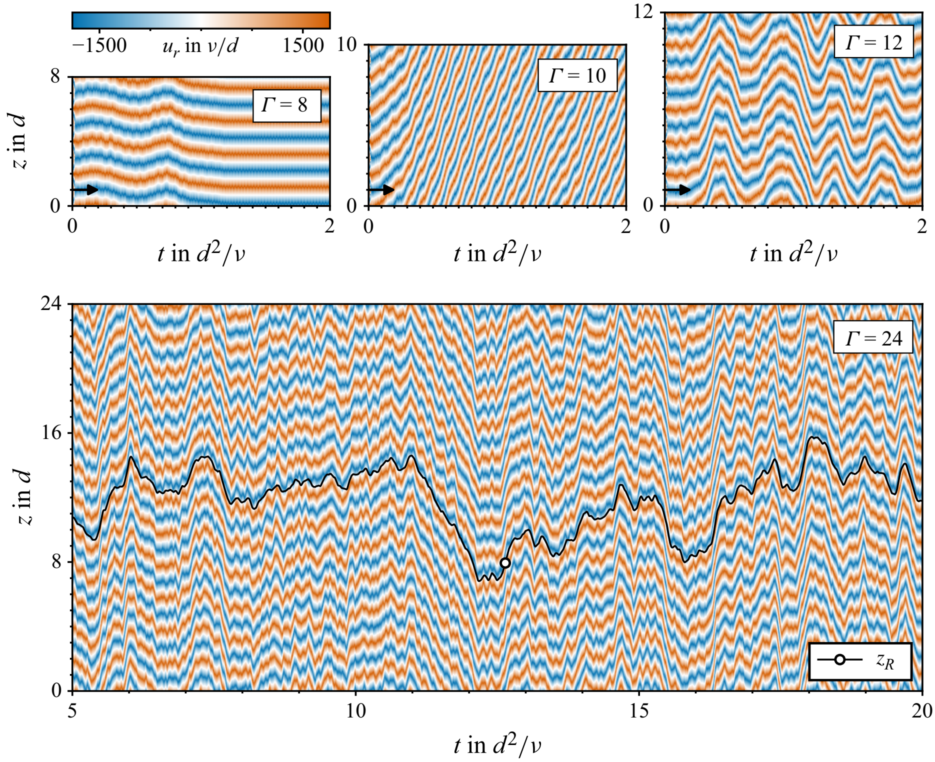

Figure 1. Temporal evolution of radial velocity along an axial line at mid-gap. The aspect ratio Γ of axial length to radial gap corresponds to the number of vortices. For $\Gamma = 8$, after an initial transient, the vortices do not move, while for $\Gamma = 10$, they move very quickly in one direction. For $\Gamma = 12$ and 24, the vortices sporadically change their direction of motion. From Feldmann & Avila (2025).

This demonstrates the interest in axisymmetric Taylor–Couette flow from a scientific point of view. However, the imposition of axisymmetry also has the great advantage of economy. Measuring diffusion coefficients of the axial drift requires extremely long times, especially if other parameters are varied as well, i.e. the number of rolls and the Reynolds numbers. Feldmann & Avila (2025) have been able to measure these diffusion coefficients because axisymmetric simulations require only a small fraction of the time that would be required to simulate the three-dimensional flow.

One might associate axial drift (motion of the phase) with axial flux (motion of fluid particles). To investigate this, Feldmann & Avila (2025) have compared simulations in which the axial flux is set to zero with those in which the net axial pressure gradient is zero. Either condition is valid for a periodic direction, but the choice has significant consequences if the flow is not reflection symmetric (e.g. Edwards *et al.* 1991). Feldmann & Avila (2025) find that in the absence of axial flux, the drift is considerably reduced, but still undergoes Brownian motion.

3. The future

Several questions are raised by this paper. The most obvious and perplexing is the reason for the abrupt threshold. Why are shorter columns tranquil and why are slightly longer columns suddenly so jittery? What physical phenomenon could be responsible for such a sharp distinction?

The second question concerns its generality. Feldmann & Avila (2025) have given convincing evidence that the rolls in other flows, such as Poiseuille flow, Rayleigh–Bénard convection and Taylor–Couette flow with no axial flux, also undergo diffusive drift. Does drift in these flows also have a length threshold? Are the threshold and the power law decay exponent the same?

The third question concerns the applicability of these axisymmetric results to the three-dimensional turbulence which naturally occurs at these high values of Reynolds or Rayleigh number. Eckhardt *et al.* (2020) suggest that some global properties of three-dimensional turbulent Taylor–Couette flow could be captured by its axisymmetric analogue. Is axial drift one of those properties? What other properties might obey this?

Declaration of interests. The authors report no conflict of interest.

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