holds, with an intervening constant, for operators between Banach spaces. Chapter 3 gives an account of this result, together with further refinements and connections with r-summing norms.

The next topics considered are the notions of trace and determinant. The fact that there are Banach spaces without the approximation property amounts to saying that the obvious way to try to define trace on the class of nuclear operators does not work. In fact, it can be shown that there is no continuous function with the properties of a "trace" on this class. In recent years, the author and others have shown that it is possible to define a trace on certain smaller operator ideals described in terms of s-numbers or eigenvalues. Somewhat similar considerations apply to determinants.

Chapters 5 and 6 are devoted to showing how the abstract machinery developed earlier can be applied to "concrete" matrix and integral operators. This has its origins in Schur's theorem of 1909, but again there has been a spate of results in the last few years. The general idea is to show how smoothness properties of a kernel translate into convergence properties of the eigenvalues. Results of this kind are described first for matrices, particularly of "Hille-Tamarkin" and "Besov" type, and analogous results are then presented for integral operators.

Finally, the author devotes a complete chapter to a historical survey. This is conducted with characteristic thoroughness, starting from Gauss and Cauchy. To test the reader's level of cultural attainment, there are extended quotations in six different languages.

In order to keep the volume self-contained, there is a certain amount of overlap with the author's earlier book *Operator Ideals*. In particular, there is an opening chapter on r-summing and (r,s)-summing operators. Before this, there is a preliminary section setting out the prerequisites. These amount mostly to fairly basic Banach space theory, except that it will be daunting for some to discover that familiarity with interpolation theory is assumed. Readers who do not possess this familiarity will in fact find that they can manage without it most of the time, and in some cases the language of "interpolation couples" could be replaced by the ordinary Hölder inequality.

This book is the first unified account of a new and exciting subject, and it contains a wealth of information for the reader who is sufficiently determined. However, it must be said that a high degree of determination is needed. One useful concession is made to readers who would like to settle for something less than the entire book: the sections pertinent to the most important results are identified. This apart, the book is not one to read casually or dip into. The material is densely packed, with a minimum of motivation and discussion. For the reviewer at least, it would be helpful if new definitions were accompanied by a few simple examples. The author relies uncompromisingly on fixed notations introduced somewhere in the book: typically, these are gothic characters surrounded by superscripts and subscripts, and the list of them runs to three double-column pages. None of this makes for easy reading. However, the primary purpose of the book is to give a full and thorough treatment of the subject area, and in this it has undoubtedly succeeded. It will surely become a standard reference.

G. J. O. JAMESON

SUNDER, V. S., *An invitation to von Neumann algebras* (Universitext, Springer-Verlag, Berlin–Heidelberg–New York, 1986), xiv + 171 pp., 3 540 96356 1, DM 68.

Rings of (Hilbert space) operators, later to become known as von Neumann algebras, were subjected to a deep and penetrating analysis by Murray and von Neumann in a series of papers from 1936 to 1943. They concentrated on the so-called factors (algebras with trivial centre), which may be regarded, via a direct integral theory, as the building blocks for more general von Neumann algebras. Factors were classified into types I_n $(1 \leq n < \infty)$, I_∞, II_1, II_∞ and III, and constructions for examples of each type were given. However, it was known that the classification was not fine enough to detect $*$-isomorphism, and it was realized that the level of complexity and mystery was an increasing function of the Roman numeral!

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Tremendous steps forward in understanding the dark world of the type III factor were made through the work of Tomita, Takesaki and Connes in the late 1960s and early 1970s. Spectral invariants, arising from the modular automorphism group of the Tomita–Takesaki theory, enabled Connes to sub-classify type III factors as type III_λ $(0 \leqq \lambda \leqq 1)$. On the other hand, Takesaki's duality theorem for crossed products, together with the modular theory once again, permitted powerful links to be established between type III algebras and the (better understood) type II algebras. These developments led on to the work of Connes, and finally Haagerup, which showed that for hyperfinite factors (inductive limits of matrix algebras) the classification I_n, I_∞, II_1, II_∞, III_λ is complete up to *-isomorphism.

In contrast with other recent books which offer a more rounded introduction to the theory of both von Neumann algebras and C*-algebras, Sunder's book aims as directly as possible for the advances outlined above (excluding the hyperfinite theory). The emphasis is on the basic ideas rather than complete detail, so that whilst most of the important results are stated in full generality, proofs are often given under certain simplifying assumptions (typically, as in the case of the Tomita–Takesaki theorem, that a particular operator is bounded). Although such assumptions may be very restrictive in practice, it seems a reasonable approach for someone wishing to acquire the flavour of this area "first time around" without being unduly daunted by too many technicalities. Even so, a solid background in analysis is very much a prerequisite. The author is careful to point out the short-cuts that are taken and generally takes care to indicate the sources that the reader should consult for further details.

The first two chapters deal with the pre-Tomita–Takesaki era. Chapter 0 reviews some basic operator theory, topologies for operator algebras and the double commutant theorem, and gives an introduction to the predual which is perhaps a little too heavily biased towards the special case of the trace class. Choosing what to include in such an introductory chapter will always be a rather difficult matter, but the functional calculus could perhaps have been treated more fully here. Chapter 1 is devoted to the basic type I, II, III-classification, with the treatment quickly specializing to the case of factors. In the comparison theory the author chooses to work with subspaces rather than projections, and this leads to some awkward notation with italic and calligraphic M's in close proximity.

The Tomita–Takesaki theory is covered in Chapter 2, with sections on weights, the KMS condition and the Radon–Nikodym theorem of Pedersen and Takesaki. Chapter 3 begins with Connes' unitary cocycle theorem on the essential uniqueness of the modular automorphism group and then proceeds via a discussion of spectral theory to the III_λ classification. Chapter 4 covers discrete and continuous crossed products, Takesaki's duality theorem and the links between type III and type II algebras, and gives constructions for factors of the various types.

A considerable number of results throughout the text are dealt with in the form of exercises. These form a sufficiently important part of the work that the reader is necessarily forced into action. However, the load is lightened by the author's engaging sense of humour which pervades the book. For example, when pointing out that a possibly unbounded operator might turn out to be bounded, he writes "when that happens, the consequent relief would, it is hoped, offset the conflict with our notational convention". Few mortals will disagree! Backed up by other texts and original sources, Sunder's book should provide a good introduction to a difficult and important area of mathematics.

R. J. ARCHBOLD

BURN, R. P., *Groups: a path to geometry* (Cambridge University Press, Cambridge) xii + 242 pp., cloth: 0 521 30037 1, 1985, £30, paper: 0 521 34793 9, 1987, £9.95,

Several authors have attempted to write books on group theory providing an alternative approach to a standard course. As well as R. P. Burn's book, another book from the Cambridge

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