

GROVE, L. C. and BENSON, C. T., *Finite reflection groups* (Graduate Texts in Mathematics 99, Springer-Verlag, Berlin-Heidelberg-New York, 2nd ed. 1985), x + 133 pp., DM 78.

There are several sources for material on finite reflection groups, most of which are quite condensed. This book gives a leisurely account and no advanced topics are required as background. The need for such a book is clear; indeed the first edition of this book, published in 1971 by Bogden and Quigley, has been in demand but out of print for some time. This book could be used either as the basis for a specialised final year undergraduate course involving some geometry that reinforces previous work on linear algebra and group theory, or as a source for methods involved in the classification of Lie algebras.

After an account of the prerequisites, the book starts with the classification of the finite groups of isometries of \mathbb{R}^2 and \mathbb{R}^3 . The treatment is quite brisk and uses more mathematics than the classic source *Symmetry* by Hermann Weyl. For motivation the student should read Weyl's book first. There follows a short chapter on fundamental regions, a topic for which there are few elementary accounts available. After treating the basics, some two-dimensional examples are given and then an indication of how to construct a fundamental region for the rotation group of the cube, but the constructions for the other groups are left as exercises and I wonder if it is realistic to expect inexperienced readers to cope with these.

At this point the book specialises to finite groups generated by reflections, that is, Coxeter groups in the terminology here. The basic material on roots is developed carefully with several examples worked out. The heart of the book is the chapter on the classification of Coxeter groups, which includes a discussion of the crystallographic condition. It contains a careful account of the construction of the irreducible groups, including a table giving explicit formulae for their root systems. All this is developed purely algebraically with the occasional diagram to illustrate a point. Following Coxeter's original paper, there is a chapter on generators and relations for Coxeter groups; a helpful feature is the use of diagrams as an aid to manipulations involving relations.

The final chapter on invariants is new to this edition and clearly adds a great deal. Firstly, it introduces invariant theory, something that few if any modern low level texts do, despite the fact that the subject is both an important and a basic one that has new and significant applications. Secondly, the material in this chapter justifies to a considerable extent the importance of Coxeter groups—they are the finite groups whose invariants form polynomial algebras. The treatment here is restricted to real groups, whereas the true significance is probably more clearly seen by considering unitary reflection groups. Nevertheless, I believe that the authors have made a sensible decision in restricting themselves to the real groups and in any case their treatment can be easily adapted to cover the more general situation.

This book is a very useful one; it gives clear proofs of interesting results; in a few cases simpler proofs can be given but this is a very minor blemish. The only drawback that I found was the lack of emphasis on motivation for studying the subject; however, some historical remarks are made, and the new final chapter has improved the position considerably in this respect. Someone who knows why he wants to study reflection groups will find this an excellent source for a significant number of interesting results and methods.

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