THE MAGNETIC FIELDS OF SINGLE WHITE DWARFS

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I. INTRODUCTION.

The first magnetic field in a white dwarf was discovered nine years ago (Kemp et al. 1970). Since that time magnetic fields have been detected in a total of 12 single white dwarfs and four white dwarfs in close binary systems. These magnetic fields have been found both through the Zeeman splitting and shifting of spectral lines and through the continuum circular (and sometimes linear) polarisation which a field may produce in the optical flux of a star in which it occurs. The fields so far detected appear to range between 3 MG and more than 100 MG. For stars with fields less than about 40 MG, both the observed Zeeman shifts of spectral lines and the observed circular polarisation appear to be reasonably well understood. For stars with fields above this level, attempts so far to identify absorption features in the spectra and to account for the continuum polarisation have not been very successful; indeed, there is considerable dispute about the fundamental mechanism producing the continuum polarisation, especially the linear component.

In this review, I shall briefly survey the work which has been done over the past few years on identifying and accounting for the observed Zeeman shifting of spectral lines, and then examine efforts which have been made to understand and model the observed polarisation spectra. I shall also consider what conclusions may be drawn about the distributions of magnetic field strength over white dwarf surfaces from models of the observational data. Finally, I shall discuss some recent work on the problem of why white dwarfs show either rather large magnetic fields (3-300 MG) or no detectable ones ($\lesssim$1 MG).

II. OBSERVATIONS.

The observational material on the 12 single white dwarfs discussed here has recently been reviewed by Angel (1977, 1978). Apart from the work mentioned there, the only new observational material known to me is a paper by Landstreet and Angel (1980) which will present intermediate resolution polarisation spectra of G195-19, G240-72, and GD229.

The present state of observations of the known single magnetic white dwarfs is summarised in Table 1. The first column gives one common designation for each star. Column (2) lists the approximate

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Table 1
Observations of Magnetic White Dwarfs (1979)

(1) Star	(2) T_e (K) $L/L_\odot$	(3) Compos. Period	(4) CP obs V_B(%)	(5) LP obs P_B(%)	(6) B_e (MG) B_s (MG)	(7) References
Feige 7	22000 $2x10^{-2}$	H,HeI v 132m	F .4v	no obs	≤4 v 18	1,2
G99-37	6200 $1x10^{-4}$	C_2,CH	GD,MC .9	F <.2	3.6	3,15
G99-47	5600 $1x10^{-4}$	H	MC .4	F <.3	3.5 15	4,5
GD90	12000 $2x10^{-3}$	H	GD <.05	no obs	0 5.5	2,5,6
G195-19	7000 $3x10^{-4}$	no lines 1.33d	MC -1 v	F <.2?		2,7,8
PG1015+01	10000:	no id, v 99m	F 1.5 v	no obs		9
LP790-29	8600	C_2	GD 8	no obs		10
BPM25114	20000 $1x10^{-2}$	H 2.84d?	F 1.5v?	no obs	8-10(v?) 25	5,14
G240-72	6000 $6x10^{-5}$	no id	MC -1.5	MC 2.5		8,11
G227-35	7000 $2x10^{-4}$	no id	MC -3	F <.2		12
Grw+70°8247	12000 $2x10^{-3}$	no id	PD,MC -4	MC 4	50? >100?	13,15
GD229	22000 $5x10^{-2}$	no id	MC -1.5	MC 3		8

1. Liebert et al. 1977. 2. Angel et al. 1980. 3. Angel and Landstreet 1974. 4. Liebert et al. 1975. 5. Wickramasinghe and Martin 1979. 6. Angel et al. 1974a. 7. Angel et al. 1972. 8. Landstreet and Angel 1980. 9. Green et al. 1980. 10. Liebert et al. 1978. 11. Angel et al. 1974b. 12. Angel et al. 1975. 13. Landstreet and Angel 1975. 14. Kemp 1977b. 15. Angel 1978.

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effective temperature and luminosity for each star for which these are known. Column (3) lists atomic or molecular species whose lines have been identified in the spectrum of each star ("no id" indicates that lines are present but not identified; "v" indicates spectrum variability) and the period of variation of variable features if known. The following two columns summarise the types of observational material available for circular and linear polarisation respectively, and list a typical continuum value for each kind of polarisation in blue light. Here "F" indicates that only broadband filter polarisation spectra are available; "MC" denotes polarisation spectra obtained with the Oke multichannel spectrophotometer, usually covering roughly the range 3500-10,000 Å with 160 Å resolution shortward of 5900 Å and 360 Å resolution longward of that wavelength (however, the resolution for Grw+70°8247 is twice that good); "GD" indicates digicon spectropolarimetry with a grating spectrograph, covering about 1000 Å in the blue with ~25A resolution; and "PD" indicates digicon observations with a prism spectrograph covering roughly 4000-7000 Å with resolution varying between 25 Å at 4000 Å and 125 Å at 7000 Å. The notation "no obs" indicates that no observations have been published; < x indicates that no polarisation is detected with an estimated upper limit of x. Column (6) lists the value of the surface averaged longitudinal magnetic field (or effective field) B_e and the mean field amplitude (mean surface field) B_s for those stars for which a reliable estimate is available. Finally, the last column lists the most recent references for the polarimetry and magnetic field models summarised in the preceding 3 columns. It may be seen from Table 1 that the observations of the known single magnetic white dwarfs are not yet in a completely satisfactory state.

III. EFFECTS OF THE MAGNETIC FIELD ON THE PHOTOSPHERE

A. SPECTRAL LINES.

It has been known for many years that the presence of a large magnetic field in a white dwarf atmosphere would cause absorption lines in the spectrum to split and become polarised due to the Zeeman effect, and the first substantial search for magnetic white dwarfs (Angel and Landstreet 1970a) was based on this effect. In fact, the first few magnetic white dwarfs discovered were identified by their continuum polarisation, but subsequently four magnetic white dwarfs (Feige 7, G99-47, GD90, and BPM25114) have been found that exhibit hydrogen spectra which are recognizable but strongly distorted by the magnetic fields. One of these, Feige 7, also shows the magnetically distorted spectrum of HeI. (G99-37 shows Zeeman polarisation in the g-band of CH at 4300 Å, but because of the relative insensitivity of molecules even to large magnetic fields, it shows an absorption spectrum little different from the one which would appear with no field). In these five stars, the location of lines in the absorption spectrum is in good accord with theory, and this furnishes a valuable means of determining field strengths for a few stars on the basis of well understood physics (see column (6) of Table 1 and the papers referred to there).

The five stars whose field strengths have been determined by modelling of the Zeeman shifted line positions and polarisations all have fields of less than 40 MG. It is generally believed that the remaining seven stars in Table 1 have magnetic fields substantially larger than this value (Angel 1978). An important basis for this conclusion is that the absorption features that are present in all the stars except G195-19 are not identifiable as transitions of H or He either in their normal positions or perturbed by fields of up to ~50 MG (see Kemic 1974b). (The one exception to this statement is LP190-29 which has a deep depression in the blue probably produced by the Swan bands of C_2). Above 50 MG, the calculations of energy level shifts (Kemic 1974a; Smith et al. 1973; Praddaude 1972) have only been done for the levels with $n \leq 3$ in hydrogen (hence only for Hα in the visible spectrum), and above 240 MG are only available for a few discrete field values (see Garstang 1977 for a review and further references). Until a better grid of calculations is available for hydrogen, and calculations are made for helium, secure identification of spectral features, and use of these features for field determination, will be uncertain for the white dwarfs with fields above 50 MG.

B. CONTINUUM POLARISATION.

Theoretical and laboratory work by Kemp (1970) and co-workers (Kemp, Swedlund, and Evans 1970) led to recognition of the fact that thermal radiation from a hot plasma threaded by a strong magnetic field ($\gtrsim 0.1$ MG) would exhibit a detectable amount of circular polarisation, and for a still stronger field ($\gtrsim 50$ MG) would be linearly polarised as well. The values of fractional circular (V) and linear (P) polarisation arising from an optically thin emitter are given by

$$V_{em}(\nu) = \alpha_1 \left(\frac{\nu_L}{\nu}\right) \cos\theta \qquad (1)$$

and

$$P_{em}(\nu) = \frac{\alpha_2}{2}\left(\frac{\nu_L}{\nu}\right)^2 \sin^2\theta \qquad (2)$$

where $\nu_L = eB/4\pi mc$ is the Larmor frequency, ν is the frequency of observation, and θ is the angle between the field direction and the line of sight. The factors α_1 and $\alpha_2/2$ are both of order unity and depend on the nature of the emitter. Table 2 gives values of α_1 and $\alpha_2/2$ for a few important cases.

Table 2. Polarisation of Various Opacity Sources.

Opacity Source	α_1	$\alpha_2/2$
H, He free-free (Kemp 1977)	4	6
H, He bound-free (Lamb and Sutherland 1974)	4	5
He^- free-free (Landstreet and Angel 1975)	2.85	2.75

Equations (1) and (2) apply only as long as $(\nu_L/\nu) = .466\, B_8 \lambda_\mu \ll 1$, where B_8 is the field in units of 10^8G and λ_μ is the observed wavelength in microns. The values of α_1 and $\alpha_2/2$ given in Table 2 are accurate only for transitions involving levels whose quadratic Zeeman shift is small compared to the linear shift.

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The asymmetry of emission rates in orthogonal polarisations represented by Equations (1) and (2) also occurs as dichroism in absorption: absorption coefficients for orthogonal polarisations are different, radiation in orthogonal polarisation states comes to us from different average depths in the stellar photosphere, and the net radiation is polarised. Utilisation of this effect led to the first detection of a magnetic field in a white dwarf when strong circular and linear polarisation was detected in the white dwarf Grw+70°8247 (Kemp et al. 1970; Angel and Landstreet 1970b).

When an accurate circular polarisation spectrum of Grw+70°8247 was obtained (Angel and Landstreet 1970b) it became clear that the polarisation did not follow the simple $V \propto \lambda$ law of Eq.(1). An explanation was found by Shipman (1971), who showed that the effects of radiative transfer on polarisation of the emerging radiation are quite important. Essentially, the decrease of the dichroism with increasing frequency is compensated in the visible by a steepening of the source function S_ν as a function of optical depth, and the circular polarisation of the emerging radiation is given approximately by

$$V(\nu) = V_{em}(\nu)\left(\frac{h\nu}{8kT}\right)\left\{\frac{1}{2}\left[1 - \exp(-h\nu/kT)\right]\left[\frac{\kappa_\nu}{\kappa} + 1\right]\right\}^{-1} \qquad (3)$$

where κ_ν is the monochromatic absorption coefficient at ν and κ is a mean absorption coefficient (Shipman 1971; Landstreet and Angel 1975). The derivation of Equation (3) involves numerous simplifications and approximations to the real situation (in particular it is assumed that S_ν is a linear function of τ, and that the magnetic field is completely uniform and aligned more or less along the line of sight). Nevertheless, it allows one to obtain some insight into the effects of radiative transfer on the situation. For stars with temperatures in the range 5000-25000 K and for visible radiation, the term in curly brackets is approximately 1 and varies only slowly with wavelength. The term $(h\nu/8kT)$ lies roughly in the range 0.1 to 0.5, so the polarisation is somewhat less than expected from the optically thin emission calculation. The diminution of the polarisation is more severe at higher temperatures. The product of the $V_{em}(\nu)$ term and the $(h\nu/8kT)$ term is independent of wavelength, so all the wavelength variation comes from the slowly varying term in curly brackets. Only in the infrared, where $h\nu/kT < 1$, does the polarisation rise with increasing wavelength in accordance with Eq.(1).

Various forms of Equation (3) have been used by Shipman (1971), Angel and Landstreet (1974), Landstreet and Angel (1975), Sazonov and Chernomordik (1975), and Brown et al. (1977) to analyse the polarisation spectra of several magnetic white dwarfs. A numerical method which is basically the same as Equation (3) but is based on more realistic model atmospheres has also been used by Shipman (1971). The essential result of these calculations is that they reproduce approximately the observed polarisation spectra of G99-47 (H-rich) and G99-37 (He-rich), and perhaps the overall circular (but not linear) polarisation spectrum of Grw+70°8247. For G99-37, the field strength determined from the continuum polarisation and from the Zeeman polarisation of the g-band of CH seem to be in good agreement (Angel and Landstreet 1974; Landstreet and Angel 1975), while for G99-47 and perhaps GD90 it appears that the field determined from the continuum polarisation is perhaps a factor of 3 or 4 smaller than that inferred from the line spectrum (Liebert, Angel and Landstreet 1975; Brown et al. 1977).

Schmid -Burgk and Wehrse (1976) have calculated the expected polarisation spectra for continuum and absorption edges of hydrogen-rich white dwarfs having magnetic fields in the range of 10 to 100 MG and temperatures between 6000 and 9000 K. Although their models still assume that the magnetic field is uniform, the treatment of the radiative transfer problem is much more precise than the approximate theory based on Eq.(3). Schmidt-Burgk and Wehrse obtain self-consistent solutions to the full set of equations of transfer for polarised light, using expressions for the various absorption coefficients based on the work of Lamb and Sutherland (1974). The results resemble those obtained from Eq.(3), but no detailed comparison of these calculations either with simpler theory or with observation has yet appeared.

Recently still more elaborate calculations of the expected flux and polarisation spectra for both lines and continua have been carried out by Martin and Wickramasinghe for modelling of the three hydrogen-rich stars in Table 1 (Martin and Wickramasinghe 1978, 1979b; Wickramasinghe and Martin 1979). Their calculations are based on accurate numerical solutions of the full set of equations of transfer for polarised light through a dichroic plane-parallel atmosphere of realistic structure. In addition, the distribution of field strength over the stellar surface is taken to be dipolar, probably a more realistic assumption than the uniform field implicitly assumed in earlier work. Martin and Wickramasinghe's results are very encouraging: reasonable agreement is found between the calculated models and observations for both flux and polarisation. The lack of agreement in earlier H-rich models between line and continuum polarisation appears to stem from the oversimplification of the approximate models. (The various methods of calculating polarisation spectra are compared by Martin and Wickramasinghe 1979c).

Thus for the stars of Table 1 for which $B \lesssim 40$ MG, our theoretical understanding seems satisfactory. Models of Feige 7 and G99-37 which are as elaborate as those calculated by Martin and Wickramasinghe for the H-rich stars will be valuable; we may have every hope that they will be successful.

Further progress on model calculations of the spectra of the higher-field stars of Table 1 is frustrated at present by lack of adequate basic physics. Several new effects occur. The approximations for circular and linear dichroism represented by Equations (1) and (2) break down as the quadratic Zeeman effect becomes important for $B \gtrsim 100$ MG. Linear polarisation becomes comparable to circular polarisation in magnitude as $\nu_L/\nu \rightarrow 1$. The cyclotron absorption resonance moves into the visible spectrum for $B \gtrsim 150$ MG. (An early and rather simple-minded prediction that white dwarfs with magnetic fields of

~200 MG would show a circular polarisation of a few percent was based on recognition of the polarising effect of cyclotron absorption smeared out by non-uniform field [Landstreet 1966]).

It has been suggested by several authors that strong Faraday rotation of emerging linearly polarised radiation would prevent the photosphere of a white dwarf with a surface field of ~100 MG from showing detectable linear polarisation due to variations in the amount of rotation from one part of visible disk to another (Sazonov and Chernomordik 1975; Rosi, Zimmerman and Kemp 1976; Ingham, Brecher and Wasserman 1976; Angel 1977). For conditions found in a typical H-rich white dwarf of $T_e = 10^4$K and B = 40 MG along the line of sight, the expression for Faraday rotation (for $\nu_L/\nu \ll 1$) is

$$\frac{d\phi}{dr} \simeq n_e e^3 B\lambda^2/2m^2c^4, \qquad (4)$$

which predicts ~10^3 radians of rotation through one optical depth of atmosphere, enough to greatly reduce the integrated linear polarisation from the star if indeed the rotation has the effect of randomising position angles of emerging linearly polarised radiation from various parts of the photosphere.

I do not believe that such destructive Faraday rotation occurs to any significant extent, even at B = 100 MG. Basically, at each point in the stellar atmosphere two orthogonally polarised eigenmodes of the radiation field will propogate independently. In general these will be elliptically polarised. At a point where the field is exactly along the line of sight the eigenmodes will be right and left-handed circularly polarised waves; where the field is exactly transverse to the line of sight the eigenmodes will be linearly polarised parallel and perpendicular to the field lines. Because the scale height of the atmosphere is so small (~10 km) to quite a good approximation we may take the field to be uniform along a line of sight from a point of negligible optical depth ($\tau \lesssim 10^{-3}$) to optical depth τ~1, so that essentially one constant pair of eigenmodes propagates along the full line of sight emerging from the atmosphere. These eigenmodes will certainly suffer differential retardation (the Faraday effect) relative to one another, but they will also be the eigenmodes exhibiting differences in absorption from one another. Thus the emerging polarisation along one line of sight will have the same character as the retardation along that line and will be unaffected by it. For example, in a transverse field, the eigenmodes are parallel and perpendicular to the field lines, and the resulting polarisation of the emerging radiation will also be parallel to or perpendicular to the field lines. The difference in propagation speed will lead to the medium acting as a linear waveplate having its principal axes parallel and perpendicular to field as well. Such a waveplate would scramble emerging circular polarisation if any were trying to emerge, but only linear polarisation parallel to one of the waveplate axes is present, and this is unaffected by the retardation. A similar situation exists for waves propagating at other angles to the field lines. (The effect of a transverse field in a plasma on circularly polarised light propagating through it is known as the Cotton-Mouton effect. For conditions considered here the magnitude of this effect is smaller than that given in Eq.(4) by ν_L/ν; this would naively lead to the prediction that for 40 MG even circular polarisation from a photosphere is strongly reduced by differential retardation. See Kraus 1966 for a simple discussion. Only if the magnetic field varies strongly in direction along the line of sight from $\tau \sim 10^{-3}$ to $\tau \sim 1$ will Faraday rotation have a significant effect.

Observational evidence also suggests that strong linear polarisation may be produced in the atmospheres of magnetic white dwarfs. Decreases in linear polarisation occur across photospheric absorption features of Grw+70°8247 (Angel 1978) and GD229 (Landstreet and Angel 1980). (Such changes also occur in circular polarisation across absorption features in G99-37, GD90, LP790-29, G240-72, Grw+70°8247, and GD229). See § IV below.

To my knowledge, the only attempts to reproduce the polarisation spectra of high-field white dwarfs with photopheric models are the studies of Grw+70°8247 (Shipman 1971; Landstreet and Angel 1975; Angel 1978), G240-72 (Martin and Wickramasinghe 1979a), and GD229 (Wickramasinghe and Martin 1978). The analysis of the polarisation of Grw+70°8247 by Landstreet and Angel (1975) is based on an extension of the standard approximate solution represented by Eq.(3) to fields so high that its applicability is dubious; nevertheless the overall shape of the circular polarisation spectrum is reproduced by a helium atmosphere at 10000-12000 K with a longitudinal field of 40-50 MG. The strong dip in circular polarisation at about 3400Å is interpreted as the effect of the 2^3P absorption edge, and the absorption line at λ5855 may be the π component of λ5876 of HeI, shifted by the quadratic Zeeman effect. The general amplitude of the observed linear polarisation requires B~100 MG, consistent with the effective field of 50 MG inferred from the circular polarisation, but the approximate theory offers no obvious explanation for the strong variation of linear polarisation with wavelength.

Recently Angel (1978) has considered more seriously the possibility that some of the absorption features in Grw+70°8247 could be due to hydrogen in a field of ~250 MG. The lack of adequate calculations of level shifts in H and He in the presence of such large fields is a great obstacle to such considerations. The models of Grw+70°8247 cannot yet be considered very satisfactory.

Wickramasinghe and Martin have taken a different approach and tried to determine the extent to which the spectra of G240-72 and GD229 may be understood by invoking fields large enough (150-400 MG) to move the cyclotron resonance into the optical, and then considering the effects of the cyclotron resonance on the spectrum and polarisation. For G240-72, (Martin and Wickramasinghe 1979a) a model having a centred dipole of polar field strength 390 MG viewed from nearly in the equatorial plane is assumed. The atmosphere structure is that of a 6000K H-rich white dwarf having a very steep steep temperature gradient at low optical depths, in which all continuum polarisation and atomic line absorption is ignored. The cyclotron resonance occurs at various wavelengths on various parts of the visible stellar disk because of the variation in field strength of the dipolar field by a

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factor of two from magnetic equator to pole. This variation in the wavelength of the cyclotron absorption line results in the overall absorption being a broad shallow depression similar to that found in G240-72. The circular polarisation spectrum is also reminiscent of the observed one, including a sign change near the middle of the depression. However, the predicted linear polarisation (this is not shown in Martin and Wickramasinghe's Figure 2, but may be inferred reasonably unambiguiously from the information in their Table 2) of about 8% in a band from 4000 to 5500Å is much larger than the observed value in this band, about 1.5% (Angel et al. 1974b; Landstreet and Angel 1980). Viewing this model from other directions reduces the linear polarisation somewhat but increases the circular polarisation; to reduce both, the temperature gradient must be flattened which makes the absorption trough shallower than is observed. Basically, the difficulty with this model in accounting for the broad absorption band in G240-72 is that it makes far too much polarisation in the absorption band when conditions are right to reproduce the observed absorption.

GD229 has strong absorption lines located at λλ4170, 5240, and 8000Å, each more than 200Å wide with the strongest (at 4170Å) about 50% deep (Greenstein and Boksenberg 1978). The main problem here is to make the field sufficiently uniform for the cyclotron resonance to appear in the spectrum as a line rather than a broad, shallow trough. Wickramasinghe and Martin (1978) suggest a model which is a weighted superposition of a completely uniform field of strength B = 260 MG plus a dipole field of equal polar field strength. They identify the λ4170 feature as the cyclotron resonance, and find that they can produce an absorption feature rather similar in appearance to the observed line with a field about 85% uniform, 15% dipole. It is perhaps a little difficult to imagine nature providing such a uniform field, but the most severe difficulty again concerns the polarisation of the absorption line. This is not given in Wickramasinghe and Martin's (1978) paper, but polarisation for a similar field distribution is tabulated in Tables 2 and 3 of Martin and Wickramasinghe (1979a). The polarisation of the cyclotron resonance line (linear or circular, depending on aspect) must be very high, of the order of tens of percent. Observations (Landstreet and Angel 1974, 1980) show no excess polarisation through the λ4170 line in either linear or circular polarisation above the continuum polarisation; in fact the circular polarisation drops to less than 0.5% in the line. It appears to me that the observed lack of strong polarisation in the absorption lines in this star is an insuperable obstacle to interpreting any of the lines as due to cyclotron resonance. At present, then, cyclotron resonance models do not seem very promising for explaining the absorption features in these two stars.

IV. MODELS WITH MAGNETIC CORONAE.

Several authors have proposed models in which the polarised radiation from the more strongly polarised white dwarfs originates as cyclotron or synchrotron radiation from electrons moving in a magnetosphere or corona around the white dwarf. The basic motives behind these models seem to be concern about the difficulty of producing linear polarisation in a white dwarf photosphere in spite of Faraday rotation (but see the discussion of this point in §IIIB), and the fact that such models easily produce sign changes in linear and circular polarisation such as the 90° rotation of linear polarisation observed in Grw+70°8247 and the changes of sign of circular polarisation found in G195-19 and G240-72. Before discussing these models individually, it is appropriate first to make a few general points concerning coronae in single magnetic white dwarfs.

(1) Because the observed polarisation in several cases approaches 10%, a corona which will contribute significantly to the observed polarisation must be very bright: $L_{cor} \gtrsim 10^{-1} L_*$. It is not clear that such a corona would not be detected optically by Hα emission, HeII lines, coronal lines, or photometric variations of an irregular sort. This point is not examined in any of the papers discussing coronal models. Certainly in the close binary systems with mass transfer onto magnetic white dwarfs (the AM Her systems) where we observe cyclotron radiation from hot ($T \gtrsim 10^5$K) gas in a strong (~100 MG) magnetic field, the "corona" is easily detected by the emission lines produced.

(2) It is not clear that there is any compelling theoretical reason to expect the existence of such a bright corona around a magnetic white dwarf. Motions in the atmosphere may generate a corona of some sort (if not suppressed by the strong magnetic field), or some mechanism may accelerate electrons to relativistic velocities, but the luminosity of such a structure may be much too low to be detectable. (The solar corona, for comparison, is about 10^{-6} times as bright as the photosphere).

(3) Observations of absorption lines in Grw+70°8247, GD229, and LP790-29 which are less polarised than the nearby continuum are not consistent with substantial continuum polarisation due to cyclotron or synchrotron radiation. The drop through an absorption line in thermal flux from a star, which is assumed to have generally low polarisation on the coronal models, would decrease the dilution of the highly polarised coronal radiation and lead to an increase in fractional polarisation through the absorption line rather than the observed decreases.

Three distinct coronal models have been discussed in the literature. Sazonov and Chernomordik (1975) consider cyclotron radiation from a geometrically thick corona (comparable in thickness to the radius of the white dwarf) in which a gas at $T \sim 10^6$K radiates in a dipole magnetic field of B~200 MG. Ingham, Brecher, and Wasserman (1976) also discuss cyclotron radiation from such a corona, but assume that the geometrical thickness of the corona is much smaller than the white dwarf radius. Finally, Rosi, Zimmerman, and Kemp (1976) consider synchrotron radiation by highly relativistic electrons moving in a dipole field of 1 to 100 MG. In this model, the electrons are trapped in a toroidal belt around the star much like a van Allen belt.

In the model of Sazonov and Chernomordik, the gas density falls off very slowly with height (the product $n_e v^2$ is assumed to vary as $r^{-\beta}$, $0 \leq \beta \leq 1$), so that radiation at a particular frequency comes from a cap-shaped surface above each magnetic pole having constant field strength and cyclotron frequency ν_c. Occultation of part of the

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emission by the white dwarf is ignored. No radiation occurs at frequencies above the value of ν_c at the poles, ν_{cp}. Because the radiating surfaces for which $\nu_c = \nu_{cp}$ is small compared to the surface for smaller ν_c, the polarised flux goes to zero at ν_{cp}. At lower frequencies, the calculated linearly polarised flux changes sign twice between ν_{cp} and $\nu_{cp}/2$. while the circularly polarised flux changes sign once. The authors compare the results of their calculation with observations of Grw+70°8247 and obtain what appear to be plausible fits over much of the visible spectrum. However, they cannot account for the observed upturn in circular polarisation for λ<3400Å, and no change of sign of circular polarisation is observed at 7000Å as predicted by the model. Furthermore, only one change of sign of linear polarisation is observed, compared to two predicted between 3400 and 6800Å. The actual mode of comparison chosen by the authors is also somewhat misleading; they calculate and plot polarised flux but the observations of Grw+70°8247 with which they compare their results are shown in percentage polarisation. Data and theory should be given in the same form for a really useful comparison.

Ingham, Brecher, and Wasserman point out that the scale height of an isothermal corona is less than about 5% of the radius of a white dwarf for $T \leq 10^8$K, so they discuss a corona which is thin compared to the white dwarf radius. This greatly changes the relative contributions from regions radiating at various frequencies between ν_{cp} (at the poles of the dipole field) and $\nu_{cp}/2$ (at the equator), and thus leads to a wavelength dependence of polarisation quite different from that found by Sazonov and Chernomordik. (In addition, the results of Ingham, Brecher, and Wasserman differ from those of Sazonov and Chernomordik in that the effect of the white dwarf in eclipsing part of the radiation is accounted for, and in adding dilution by thermal radiation to the final results). In spite of these differences, however, the main results are similar: for a dipole field distribution circular polarisation changes sign once, and linear twice, between ν_{cp} and $\nu_{cp}/2$. It thus appears that cyclotron radiation is altogether <u>too</u> effective at producing changes of sign of polarisation, and for this reason it seems unlikely to form the basis for successful models of high field stars, none of which show so many sign changes of polarisation per octave.

The results of Rosi, Zimmerman, and Kemp for synchrotron radiation by ultrarelativistic electrons in a rather smaller field are qualitatively different from those for the cyclotron radiation models. In the synchrotron model the electrons are trapped on a surface defined by the set of field lines crossing the magnetic equator at some fixed distance r_{eo} from the centre of the star. The electrons are reflected at mirror points near the poles. (Occultation by the star of part of the radiation is again ignored in this calculation but is not expected to be too important). The resulting wavelength dependance of polarisation is quite different from that produced by the cyclotron models. For a distribution of electrons which extends to the poles of the star, only one sign change in linear polarisation is predicted in the roughly two decades over which the synchrotron radiation is strong. No sign change is expected for circular polarisation. For an electron distribution more strongly confined to the star's magnetic equatorial plane, both linear and circular polarisation change sign once, but the reversal in circular polarisation is very weak. Comparison of this model is again made to the data for Grw+70°8247; to avoid the dilution problem these authors simply compare theory with observation for the ratio of linear to circular polarisation. A plausible fit is obtained (without of course being able to account for the fine structure of the polarisation spectra), but no attempt is made to fit linear or circular polarisation spectra separately. For this model, the surface field on Grw+70°8247 is expected to be less than 100 MG; it is not clear then how to account for the observed absorption features since Kemic (1974b) was unable to identify them with Zeeman shifted features of either hydrogen or helium for fields of less than 100 MG.

V. THE SURFACE MAGNETIC FIELD DISTRIBUTION.

The next question to consider is whether one can say anything yet about the distribution of magnetic field strength over the surface of any of the magnetic white dwarfs. By analogy with the main sequence stars, we might expect to find dipolar or roughly dipolar field distributions (such as quadrupoles). The dipole field is the mode of longest decay time (Fontaine, Thomas, and Van Horn 1973).

This general view may be subjected to a few observational tests. Perhaps the most convincing evidence that at least a few white dwarf magnetic fields are dipole-like comes from the fact that there is a fairly large ratio of longitudinal to surface field in two of the three stars in which they are measured independently, Feige 7 and G99-47. (GD90 shows essentially no longitudinal field, but this may well be an aspect effect). In a multipole expansion of a surface field, only the dipole component can produce a longitudinal field component of more than a few percent of the size of the surface field (Schwarzschild 1950). (This, of course, only shows that a major field component is roughly uniform, not that it is closely modelled by a centred or decentred dipole distribution). This overall uniformity is also supported by the large degree of linear polarisation found in Grw+70°8247, G240-27, and GD229: a complex field distribution would probably not result in much net linear polarisation. Finally, sinusoidal variation of circular polarisation observed for Feige 7, for PG1015+01, and for G195-19 shortward of 5500Å suggests that a major field component is dipolar.

To test the dipolar field distribution more stringently one requires detailed calculations of the sort reported by Wickramasinghe and Martin (1979), in which the flux and polarisation spectra are computed in detail for a grid of models, which are then compared to all the available data. The results so far indicate that satisfactory fits to the data can be obtained for G99-47, GD90 and BPM25114. Still uncertain is how sensitive these fits are to variations in the assumed field distribution. Could a satisfactory fit be obtained with a completely uniform field, or with a strongly decentred dipole, for example?

A few pieces of evidence may point to some type of field distribution other than dipolar. If the cyclotron absorption model of Wick-

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ramasinghe and Martin (1978) is correct for GD229, this would require the existence of a very uniform field in at least one star. However, it has been argued in §IIIA that this model is probably fundamentally incorrect because of its inability to produce unpolarised absorption lines.

The case of G195-19 is more puzzling. In this star, the circular polarisation variation in blue (λ<5500Å) light is quite sinusoidal, but the circular polarisation variation in the red (λ>6000Å) is both out of phase with the blue variation and somewhat non-sinusoidal. The only model of this phenomenon so far is one proposed by Landi Degl' Innocenti (1976), involving an oblique dipole rotator having B~1-10 MG, which produces all the blue polarisation and contributes to the red polarisation, plus a small spot with a field B~200 MG that is emitting cyclotron radiation only for λ>6000Å. On this model, the polarisation curve without the spot in red light would resemble that seen in blue light in shape and phasing, but the appearance of the high-field region adds a distortion to the original sinusoidal variation.

It is difficult to assess how plausible this model is. It requires an extraordinarily bright high-field spot ($L_{spot} \sim 10^{-2} L_*$). It makes quite specific predictions about linear polarisation which need to be tested. Detailed circular polarisation spectra at several phases (Landstreet and Angel 1980) show that the blue polarisation has quite complex wavelength dependence, more than predicted by simple continuum polarisation theory, but whether this conflicts with Landi Degl'Innocenti's model is not clear.

At present, then, almost all the data are consistent with a centred or slightly decentred dipole model giving a rather good representation of the actual field distribution, but one cannot exclude the possibility that some other qualitatively similar field distribution would do as well. Only G195-19 seems really hard to understand in this context.

VI. THE DISTRIBUTION OF FIELD STRENGTHS OVER THE WHITE DWARF SAMPLE.

Angel, Landstreet, and Borra (1980) have examined the problem of determining the probability of finding magnetic fields of various strengths in a sample of white dwarfs. Their conclusion is quite remarkable. Let P(B)dB be the normalised probability of finding a surface field in the interval dB, so that BP(B) is the probability of finding a surface field between B and 2B (i.e., the probability per octave). The value of BP(B) is found to be roughly constant at about 0.5-1% per octave all the way from ~3 to ~300 MG. In other words, one is just as likely to find a field between 100 and 200 MG as between 3 and 6 MG. This result is in striking contrast to the situation for magnetic Ap stars, where the probability per octave falls rapidly above a few hundred gauss. On the other hand, no fields at all have been detected below 3 MG and the upper limit on the probability per octave at about 1 MG is not significantly higher than the observed value of BP(B) for larger fields. One thus has the situation that almost all white dwarfs exhibit fields of less than 1 MG while a small

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fraction (3-5%) show fields above 3 MG, with any octave up to roughly 300 MG equally likely. This is a most peculiar distribution. It may even have two peaks, one at low field values and one at high fields, but this is not certain - it may simply have a long flat tail.

Recently Moss (1979) has discussed how such a situation might have come about. He has constructed theoretical models of the field structure of rotating magnetic white dwarfs with dipole-like fields parallel to the rotation axis, taking account of the tendency of rapid rotation to produce meridional circulation currents that tend to bury the surface field. He finds that for a flux of about 6×10^{23} G-cm^2 threading the equatorial plane of a star, the surface (observable) field will be about 3 MG as long as the rotational velocity of the star is below $v \sim 20$ km/sec. As v passes this critical value, the meridional circulation overwhelms the surface field and the observable field strength drops to very low values. (For other values of flux, the critical velocity is proportional to the zero-velocity surface field).

Moss points out that this result may offer an explanation of the fact that fields of say 0.1 up to 3 MG are not common enough for one to have been found yet. He argues that if in fact the probability of finding a particular flux threading a white dwarf does increase strongly for smaller values of flux, as one would expect, while at the same time most white dwarfs have rotational velocities of the order of 20 km/sec (the observational upper limits are roughly 30-50 km/sec), then the detection of strong fields ($\gtrsim 5$ MG) would not be appreciably hindered, while weak fields ($\lesssim 3$ MG without rotation) would tend to be made much weaker by the action of the circulation currents, and hence rendered undetectable.

This is an interesting hypothesis, and one which may be testable, as Moss points out, by a slight improvement in the accuracy with which white dwarf rotational velocities may be measured (not a trivial observational task, by the way). The main problems with this hypothesis are twofold, I think: first, since white dwarfs don't typically have rotational velocities of ~200 km/sec, the hypothesis doesn't account for the flatness of the BP(B) distribution in the range 3 to 300 MG, which in my opinion is the most peculiar aspect of the distribution. Secondly, the hypothesis basically replaces one very strange distribution (of surface field) with another (a rotational velocity distribution very sharply peaked at around 20 km/sec). It is not clear which situation would be harder to account for.

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