J. Fluid Mech. (2024), *vol.* 1000, F5, doi:10.1017/jfm.2024.839

JFM FOCUS ON FLUIDS

Unveiling the mystery of effective slip

Ke-Qing Xia†

Center for Complex Flows and Soft Matter Research, Department of Mechanics and Aerospace Engineering and Department of Physics, Southern University of Science and Technology, Shenzhen 518055, PR China

(Received 1 July 2024; accepted 30 August 2024)

Boundary layers are present in many natural and industrial fluid flows. The concept of boundary layers can be traced back to Leonardo da Vinci's paintings of pipe flow, where he was aware of a higher velocity away from the walls. During the 19th century, the physics of boundary conditions had been extensively debated, and the well-known Maxwell–Navier slip length was proposed in 1823. In most cases, the no-slip boundary condition is valid at a fluid–solid interface. However, with the advancement of measurement techniques, slip lengths ranging from nanometre to micrometre scales were experimentally measured, raising questions regarding the applicability of the no-slip condition. In 2003, Lauga & Stone (*J. Fluid Mech.*, vol. 489, 2003, pp. 55–77) proposed a simple model to elucidate the effect of surface heterogeneities on the slip length, elegantly bridging the microscopic structure of the wall-boundary conditions to the macroscopic effective slip length.

Key words: boundary layer structure

1. Introduction

Leonardo da Vinci (1452–1519), a luminary of the Renaissance era, was renowned not only as a painter and draftsman, but also as an engineer and fluid mechanician. Based on observations, Leonardo had illustrated many valuable concepts in modern fluid mechanics such as turbulence, the hydraulic jump and the no-slip condition (Marusic & Broomhall 2021). A notable example is the depiction of pipe flow shown in figure 1(*a*), where Leonardo commented: 'The water that rises through a pipe, that which rises highest will be furthest away from the walls of the pipe'. This comment suggested that he had been aware of the no-slip boundary condition, and it could also be found in his other works, such as the drawing of an experimental water tank in figure 1(*b*).

Although the boundary layer had been demonstrated by Leonardo, scientific inquiry of the boundary layer had to wait until the 18th and 19th centuries, when many renowned physicists started to consider this topic, including Bernoulli, Euler, Coulomb, Darcy, Navier, Helmholtz, Poisson, Poiseuille, Stokes, Hagen, Couette, Maxwell, Prandtl and

† Email address for correspondence: xiakq@sustech.edu.cn

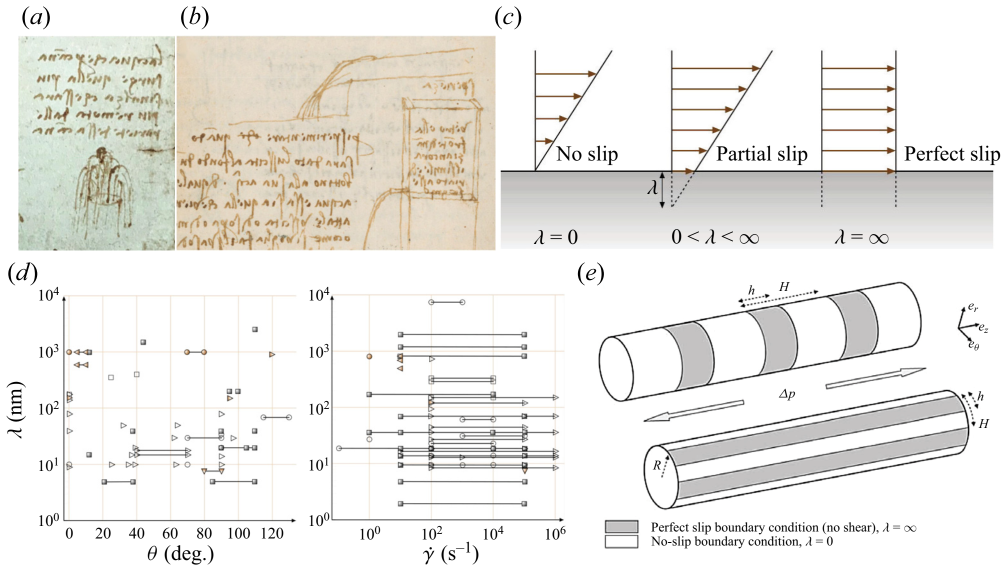

Figure 1. Leonardo da Vinci's depictions of (*a*) pipe flow (Royal Collection at Windsor, RCIN 919117r) and (*b*) water tank (Paris Manuscript I, f. 41v), both adapted from Marusic & Broomhall (2021), from where the figures are taken with permission. (*c*) Illustration of the Maxwell–Navier slip length λ and (*d*) experimentally determined slip length λ for different surface (the contact angle θ) and flow conditions (shear rate $\dot{\gamma}$), respectively, both adapted from Lauga *et al.* (2007). (*e*) Schematic drawing of the effective slip length model, adapted from Lauga & Stone (2003).

Taylor (Lauga, Brenner & Stone 2007). In 1823, Navier proposed the linear boundary condition (Navier 1823), as depicted in figure 1(*c*), where λ is the slip length. Afterwards, Maxwell quantified the slip length of gas flowing past a solid surface (Maxwell 1879), which indicated that the slip length was of the order of the mean free path of the fluid.

The definition of the boundary condition proposed by Navier and Maxwell has been widely adopted in characterising the slip properties at a fluid–solid interface. In most cases, the no-slip boundary condition (i.e. $\lambda = 0$) is found to be valid. Nevertheless, the development of the surface force apparatus enables a much more precise measurement of the slip length, down to a few nanometres. From these measurements the slip length λ is found to vary from nanometre to micrometre scales (Lauga *et al.* 2007). As shown in figure 1(*d*), λ depends on the surface roughness, surface nanobubbles, wetting properties (e.g. the contact angle θ) and the shear rate $\dot{\gamma}$.

According to these experimental observations, the principal question concerns the applicability of the no-slip condition at a solid–liquid boundary, especially for complex boundaries. To better understand the slip boundary condition, Lauga & Stone (2003) proposed an effective slip model, as illustrated in figure 1(*e*), emphasising the effect of surface heterogeneities induced by surface nanobubbles.

2. Overview

The paper by Lauga & Stone (2003) considered two elementary configurations of surface heterogeneities in steady pressure-driven Stokes flow, as illustrated in figure 1(*e*). The pipe of radius R with no-slip boundary condition (i.e. $\lambda = 0$ at a fluid–solid interface) was patterned with perfect slip boundary condition (i.e. $\lambda = \infty$ at a fluid–gas interface) of

Longitudinal case ($\lambda_{eff,\parallel}/R$)		Transverse case ($\lambda_{eff,\perp}/R$)	
Expression	Asymptotic condition	Expression	Asymptotic condition
$\ln[\sec(\delta\pi/2)] \cdot L/\pi$	All δ and $L < 2\pi$	$\delta/4$	$\delta \to 0$ and L fixed
$L\pi\delta^2/4$	$\delta \to 0$ and L fixed	$(1-\delta)^{-1}/4$	$\delta \to 1$ and L fixed
$-\ln(1-\delta) \cdot L/\pi$	$\delta \to 1$ and L fixed	$\ln[\sec(\delta\pi/2)] \cdot L/2\pi$	$L \to 0$ and δ fixed
—	—	$\delta(1-\delta)^{-1}/4$	$L \to +\infty$ and δ fixed

Table 1. Summary of the effective slip length λ_{eff} proposed by Lauga & Stone (2003).

width h and separation H in transverse and longitudinal configurations, respectively. The dimensionless spacing and percentage of the perfect slip domains are defined as $L = H/R$ and $\delta = h/H$.

The equations of incompressible Stokes flow in a circular pipe were solved numerically with the appropriate boundary conditions. Utilising analytical derivation and asymptotic approximation, the effective slip length λ_{eff} in the longitudinal ($\lambda_{eff,\parallel}$) and transverse ($\lambda_{eff,\perp}$) directions along the pipe can be derived, as shown in table 1.

When the percentage of slip is small ($\delta \to 0$), the transverse patterns can lead to a larger effective slip length than the longitudinal patterns, $\lambda_{eff,\perp} > \lambda_{eff,\parallel}$. When the percentage of slip is large ($\delta \to 1$), the longitudinal patterns have a much more significant impact on the friction than the transverse patterns. Moreover, when the distance between the perfect slip domains is small ($L \to 0$), the slip regions are approximately two-dimensional and the shear in the longitudinal case is expected to be twice as large as the shear in the transverse case. Consequently, the slip length in the longitudinal case is double that in the transverse case, $\lambda_{eff,\parallel} = 2\lambda_{eff,\perp}$.

In reality, surface heterogeneities (or slip/non-slip regions) are unlikely to be distributed in either purely transverse or purely longitudinal fashion, but may be viewed as a combination of the two. Therefore, the experimentally obtained effective slip length would be intermediate between that given by a distribution of transverse slip domains $\lambda_{eff,\perp}$ and that given by a distribution of longitudinal domains $\lambda_{eff,\parallel}$. This reflects the power of the model. It is simple yet applicable to a myriad of complicated situations. Indeed, with regard to experimentally determined slip length, the effective slip model provides a robust interpretation of the observed slip length, ensuring the applicability of the no-slip condition at a fluid–solid interface. This simple model can reproduce the sizes of slip domains that are consistent with the experimental observation of nanobubbles on hydrophobic surfaces (Tyrrell & Attard 2001) and surface roughness (Watanabe, Udagawa & Udagawa 1999).

3. Impact

The work by Lauga & Stone (2003) elegantly demonstrates how surface heterogeneities at the microscopic level affect the macroscopic effective slip length, and has garnered widespread recognition in the field of fluid dynamics. From a physical perspective, the concept of effective slip lays down the fundamental understanding of slip boundary conditions, especially over complex and heterogeneous boundaries.

From a practical perspective, given the inherent heterogeneity and the complex interplay of various physical parameters within experimental set-ups, the concept of effective slip offers a more coherent framework for guiding the design of surfaces, such as nanofluidics (Kavokine, Netz & Bocquet 2021), microfluidics (Stone, Stroock & Ajdari

2004), superhydrophobic surfaces (Quéré 2008) and liquid-infused surfaces (Hardt & McHale 2022). The slip characteristic can significantly affect fluid behaviours near the fluid–solid interface, and further affect global characteristics, including drop rebounce (Sprittles 2024), drag reduction and enhanced mixing (Rothstein 2010).

4. Outlook

Since the publication of Lauga & Stone (2003), our understanding of the slip boundary condition over the past two decades has significantly advanced, which is partly owing to the swift progression of experimental and computational fluid dynamics (Vega-Sánchez *et al.* 2022; Hadjiconstantinou 2024) as well as the widespread existence and stability of surface nanobubbles (Lohse & Zhang 2015), which were among the situations that motivated the Lauga and Stone model. From both fundamental and applied points of view, much future work remains to be done that should include multicomponent (Lyu *et al.* 2021), multiphase, phase transition (Canale *et al.* 2019), extreme conditions, molecular scale (Hadjiconstantinou 2024) and fluid–elastic interactions (Rallabandi 2024). It should also be stressed that the effective slip length developed in the Lauga and Stone model is very different from the microscopic slip length at the scale of individual molecules, which must be determined using molecular dynamics approaches and depends on material properties. A truly microscopic model based on a first-principles approach would be necessary to fully understand the slippery boundary problem. However, this has been a long-standing challenge due to the wide separation of length scales necessary to tackle this problem. In this context, the Lauga and Stone model provides a simple yet powerful framework for bridging the microscopic and the macroscopic.

Acknowledgements. The author is indebted to C. Sun for many illuminating discussions and for his help with drafting of the manuscript.

Funding. The author acknowledges support by the National Natural Science Foundation of China under grant nos 12072144 and 12232010.

Declaration of interests. The author reports no conflict of interest.

Author ORCID.
Ke-Qing Xia https://orcid.org/0000-0001-5093-9014.

REFERENCES

CANALE, L., COMTET, J., NIGUÈS, A., COHEN, C., CLANET, C., SIRIA, A. & BOCQUET, L. 2019 Nanorheology of interfacial water during ice gliding. *Phys. Rev.* X **9** (4), 041025.

HADJICONSTANTINOU, N.G. 2024 Molecular mechanics of liquid and gas slip flow. *Annu. Rev. Fluid Mech.* **56**, 435–461.

HARDT, S. & MCHALE, G. 2022 Flow and drop transport along liquid-infused surfaces. *Annu. Rev. Fluid Mech.* **54**, 83–104.

KAVOKINE, N., NETZ, R.R. & BOCQUET, L. 2021 Fluids at the nanoscale: from continuum to subcontinuum transport. *Annu. Rev. Fluid Mech.* **53**, 377–410.

LAUGA, E., BRENNER, M.P. & STONE, H.A. 2007 Microfluidics: the no-slip boundary condition. In *Handbook of Experimental Fluid Mechanics* (ed. C. Tropea, A. Yarin & J.F. Foss), pp. 1219–1240. Springer.

LAUGA, E. & STONE, H.A. 2003 Effective slip in pressure-driven Stokes flow. *J. Fluid Mech.* **489**, 55–77.

LOHSE, D. & ZHANG, X.H. 2015 Surface nanobubbles and nanodroplets. *Rev. Mod. Phys.* **87** (3), 981.

LYU, S.J., TAN, H.S., WAKATA, Y., YANG, X.J., LAW, C.K., LOHSE, D. & SUN, C. 2021 On explosive boiling of a multicomponent leidenfrost drop. *Proc. Natl Acad. Sci. USA* **118** (2), e2016107118.

MARUSIC, I. & BROOMHALL, S. 2021 Leonardo da Vinci and fluid mechanics. *Annu. Rev. Fluid Mech.* **53**, 1–25.

MAXWELL, J.C. 1879 On stresses in rarified gases arising from inequalities of temperature. *Phil. Trans. R. Soc. Lond.* **170**, 231–256.
NAVIER, C.L.M.H. 1823 Memoire sur les lois du mouvement des fluides. *Mem. Acad. R. Sci. Inst. France* **6**, 389–440.
QUÉRÉ, D. 2008 Wetting and roughness. *Annu. Rev. Mater. Res.* **38**, 71–99.
RALLABANDI, B. 2024 Fluid-elastic interactions near contact at low Reynolds number. *Annu. Rev. Fluid Mech.* **56**, 491–519.
ROTHSTEIN, J.P. 2010 Slip on superhydrophobic surfaces. *Annu. Rev. Fluid Mech.* **42**, 89–109.
SPRITTLES, J.E. 2024 Gas microfilms in droplet dynamics: when do drops bounce? *Annu. Rev. Fluid Mech.* **56**, 91–118.
STONE, H.A., STROOCK, A.D. & AJDARI, A. 2004 Engineering flows in small devices: microfluidics toward a lab-on-a-chip. *Annu. Rev. Fluid Mech.* **36**, 381–411.
TYRRELL, J.W.G. & ATTARD, P. 2001 Images of nanobubbles on hydrophobic surfaces and their interactions. *Phys. Rev. Lett.* **87** (17), 176104.
VEGA-SÁNCHEZ, C., PEPPOU-CHAPMAN, S., ZHU, L.W. & NETO, C. 2022 Nanobubbles explain the large slip observed on lubricant-infused surfaces. *Nat. Commun.* **13** (1), 351.
WATANABE, K., UDAGAWA, Y. & UDAGAWA, H. 1999 Drag reduction of Newtonian fluid in a circular pipe with a highly water-repellent wall. *J. Fluid Mech.* **381**, 225–238.