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THE STRING OF NETS

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Abstract After summarizing from previous papers the definitions of the concepts associated with nets, i.e. triples of 6-transpositions in the Monster up to braiding, we give some results.

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1. Introduction

In [**10**] the concept of a *net* was introduced and motivated as a means of studying the Fischer–Griess Monster $\mathbb{M}$. This concept had previously been alluded to in [**7–9**]; in the first of these references, no name had been given to the concept, but in the other two the word 'football', which 'net' is intended to supersede, was used. This concept has also been studied in a more general context by Hsu [**2**, **4–6**] under the name 'quilt'; note that we intend the word 'net' to be used exclusively in connection with the Monster.

We start by stating some properties of the Monster. $\mathbb{M}$ is the largest of the 26 sporadic simple groups, with order $2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$. It has 194 conjugacy classes, consisting of 150 rational classes and 22 pairs of classes linked by inversion, so that all character values are either rational or complex quadratic integers. Its smallest non-trivial representation over the complex numbers has dimension 196 883; this can be written over the rationals and therefore supports an inner product $(\cdot,\cdot)$. It also supports an algebra that is unique up to scalar multiplication and commutative but not associative; if the algebra product is written as $*$, then the triple product $(A * B, C)$ is totally symmetric in A, B and C.

Particularly relevant for net theory is that $\mathbb{M}$ is a 6-*transposition group*, i.e. it has a conjugacy class of involutions (which we call transpositions)—the class labelled 2A in Atlas [**3**] notation—such that the product of any two transpositions has order at most 6. In fact, there are just nine conjugacy classes to which such a product can belong, with Atlas labels 1A, 2A, 2B, 3A, 3C, 4A, 4B, 5A and 6A, where in accordance with

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Atlas conventions the numerical part of the label gives the order of the elements in the conjugacy class.

The centralizer of a transposition is the double cover of the Baby Monster sporadic group, $2 \cdot B$. The permutation representation of $\mathbb{M}$ over $2 \cdot B$ has rank 9, and the nine irreducible components of its permutation character include the trivial character and the one with degree 196 883. The last means that there are vectors in the 196 883-space that correspond to each transposition, and these are unique up to scalar multiplication.

It is in fact more convenient to work not in this 196 883-space but in the 196 884-space obtained by adjoining the trivial representation. We may then define a vector 1 in the trivial representation, and a vector A corresponding to each transposition $\boldsymbol{a}$, such that $(1, 1) = 3$, $(A, 1) = 4$, $(A, A) = 256$, and take the unique algebra over the 196 884-space which has 1 as an identity, in which the triple product $(A * B, C)$ is totally symmetric, and where $A * A = 64A$ for any transposition $\boldsymbol{a}$. We call this algebra the *Griess–Conway* algebra, as it was constructed by Conway [**1**] in a simplification of Griess's original construction of $\mathbb{M}$. We call the above vectors A, and certain vectors that correspond to elements of conjugacy class 3A, 4A and 5A in the Monster, *axis vectors*. Many properties of the Griess–Conway algebra are given in [**8**]. Note that our notation differs from that used in [**1**] and in Atlas in that we have doubled all inner products to eliminate fractions.

2. Nets and folded nets

This section reproduces the results of [**11**].

Consider the set of (ordered) triples of transpositions in $\mathbb{M}$. Let one such triple be $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$. We define two operations $x\colon (\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) \mapsto (\boldsymbol{b}, \boldsymbol{a}^{\boldsymbol{b}}, \boldsymbol{c})$ and $y\colon (\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) \mapsto (\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{b}^{\boldsymbol{c}})$; then x and y satisfy the relation $xyx = yxy$, which defines a familiar presentation of the 3-string braid group. Indeed, we may think of x as an operation that passes a string corresponding to $\boldsymbol{a}$ under one corresponding to $\boldsymbol{b}$, while y passes the string corresponding to $\boldsymbol{b}$ under one corresponding to $\boldsymbol{c}$.

We may also formulate the braid group in terms of the generators $s\colon (\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) \mapsto (\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{a}^{\boldsymbol{bc}})$ and $t\colon (\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) \mapsto (\boldsymbol{c}, \boldsymbol{b}^{\boldsymbol{c}}, \boldsymbol{a}^{\boldsymbol{bc}})$. As $s = xy$, $t = xyx = yxy$, $x = s^{-1}t$ and $y = ts^{-1}$, it is clear that $\langle s, t\rangle = \langle x, y\rangle$. It is easily seen that the element $z = s^3 = t^2$, which is central in $\langle s, t\rangle$, corresponds to conjugation by $\boldsymbol{abc}$, an element which is invariant under both s and t; this gives rise to another familiar presentation of the braid group as $\langle s, t \mid s^3 = t^2\rangle$.

We now define a *net* as a connected geometry associated with a subset of the orbits of z on the full set of triples. These are actually the flags of the geometry, which has rank 3. If we call the elements of the geometry vertices (corresponding to orbits under $\langle s\rangle$), edges (corresponding to orbits under $\langle t\rangle$) and faces (corresponding to orbits under $\langle x, z\rangle$), then the net will be like a polyhedron. Two elements of different types are incident if the two corresponding orbits share a flag; if they share more than one flag, we regard them as being multiply incident.

As $s^3 = z$, each vertex corresponds to three or one flags. In the latter case we say that the vertex is *collapsed*. Similarly, an edge will correspond to two or one flags, and in the latter case we say that the edge is collapsed. If a face has n flags, we call it an n-gon,

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with the usual specializations for particular values of n; n will always divide the order of $\boldsymbol{ab}$, which we call the *order* of the face, and if they are unequal we describe the face as collapsed and call n its *collapsed order*.

Allowing for multiple incidences, the number of incidences between a particular element and elements of either of the other two types will be equal to the number of flags corresponding to that particular element. In particular, a collapsed vertex or edge will be incident with just one element of either of the other two types, and an uncollapsed vertex or edge will be incident with three or two elements of either of the other two types. If there are no collapsed vertices or edges, then the geometry corresponds to an (orientable) trivalent polyhedron, where 'going round' a vertex, edge or face corresponds to applying s, t or x, respectively; in other cases we may consider the geometry to have one or more degenerate vertices or edges.

Any triple lies in a unique net; for brevity we use the phrase 'the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$' to mean the net that the triple $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ lies in.

We use the word *string* as the collective noun for the set of all (conjugacy classes of) nets—or folded nets, to be defined shortly. Hence the title of this paper.

The *Euler characteristic* of a net can be defined as $V - E + F$, where V, E and F are, respectively, the numbers of vertices, *non-collapsed* edges and faces. A familiar argument using induction on the size of a net can be used to show that, with this definition, the value of its Euler characteristic depends only on the topology of the surface defined by the union of closed discs corresponding to the faces, where the intersection of two such discs is determined by the edges and vertices common to the two relevant faces. In particular, if the surface has genus 0 or 1, the Euler characteristic of the net will be 2 or 0 respectively.

Induction can also be used to show that the Euler characteristic is one-sixth of the *defect* of the net, defined as the sum of the defects of its elements, where a non-collapsed vertex or edge has defect 0, a collapsed vertex has defect 4, a collapsed edge has defect 3, and an n-gon (whether collapsed or otherwise) has defect $6 - n$. It follows immediately from the 6-transposition property of $\mathbb{M}$ that the defect of a net will always be non-negative, so that its genus will be 0 or 1 (and its defect 12 or 0, respectively). We use the terms *netball* and *honeycomb* for nets of genus 0 and 1, respectively (in the latter case because all faces of a net with defect 0 must clearly be hexagons).

Above we defined the order of the face corresponding to the triple $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ to be the order of $\boldsymbol{ab}$. Similarly, we may define the *class* of the face to be the conjugacy class of $\boldsymbol{ab}$, and the *order* and *class* of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ to be the order and conjugacy class of $\boldsymbol{abc}$.

We now define the concept of a *folded net*. This is like a net except that we quotient out the triples of transpositions $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ by conjugation, not just by powers of $\boldsymbol{abc}$, but by the entire Monster. In other words, a folded net can be obtained from a net by quotienting out those symmetries that correspond to conjugation by elements of the Monster.

It is clear that the number of conjugacy classes of nets is the same as the number of folded nets: both are easily seen to be equal to the number of orbits on triples of transpositions of the compositum of the braid group and the group of conjugations by elements of $\mathbb{M}$.

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It is also clear that the concepts of Euler characteristic and defect for nets pass through to folded nets unchanged, and that the folded net corresponding to a netball will have genus 0. The folded net corresponding to a honeycomb may have genus either 0 or 1, but all known honeycombs fold to surfaces of genus 0, and the arguments of [**10**] suggest that there may be no counter-examples. We therefore make the following conjecture.

Conjecture 2.1. *All folded nets have genus* 0.

The following summarizes the results proved by character theory in [**11**]. The calculations, and others referred to in this paper, were done using the GAP system developed at Aachen [**12**].

Theorem 2.2. *The string of nets has* 1 400 384 *flags,* 683 *collapsed vertices and* 5000 *collapsed edges; there are* 9 *faces of class* 1A*,* 72 *faces of class* 2A*, of which* 39 *are uncollapsed (i.e. 2-gons) and* 33 *collapse to 1-gons,* 68 *2-gons and* 47 *1-gons of class* 2B*,* 311 *triangles and* 50 *1-gons of class* 3A*,* 676 *triangles and* 6 *1-gons of class* 3C*,* 6730 *squares,* 511 *2-gons and* 185 *1-gons of class* 4A*,* 6905 *squares,* 131 *2-gons and* 45 *1-gons of class* 4B*,* 88 193 *pentagons and* 144 *1-gons of class* 5A*, and* 146 957 *hexagons,* 5430 *triangles,* 747 *2-gons and* 365 *1-gons of class* 6A*.*

Theorem 2.3. *The number of folded nets with genus* 0 *is* 13 575*. Therefore, if we assume that Conjecture 2.1 holds, this is also the total number of folded nets, or the number of conjugacy classes of nets.*

3. Net terminology

In this section we reproduce definitions from [**10**] of the remaining terminology associated with nets.

(1) The *modulus* of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is the smallest positive integer n such that $(\boldsymbol{abc})^n$ commutes with each of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. If the modulus is equal to the order we call the net *faithful*.

(2) The *exponent* of a vertex, edge or face of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is essentially the power of $\boldsymbol{abc}$ by which one conjugates when going all the way round it, i.e. when applying to $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ the operation s^{-1}, t, x^{-1} repeatedly until one returns to the original flag. This defines an integer modulo m, where m is the modulus of the net. We choose the following representatives of the relevant congruence classes: the exponent of a vertex is -1 unless it collapses, in which case it is $(m-1)/3$ or $(2m-1)/3$, whichever is an integer. The exponent of an edge is 1 if it does not collapse and $(m+1)/2$ if it does. The exponent of a face is 0 if it does not collapse and a positive integer less than m if it does.

(3) A net is *small* if its exponent, i.e. the sum of the exponents of all its elements, is equal to the modulus, and it is a netball. It can be seen that the exponent is always positive and a multiple of the modulus. If a net is not small, we call it *large*. A net is small or large according to whether its exponent is equal to or greater than the

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product of the modulus and the genus. A conjugacy class is *perfect* if all nets of that class are small.

(4) The *net rotation group* of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is the set of orientation-preserving automorphisms of the net realized by elements of $\mathbb{M}$. (The net may have other automorphisms that correspond to outer automorphisms of the *net group* $\langle \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \rangle$ not in $\mathbb{M}$, or that do not correspond to a group action at all.) The *reflection* of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is the net $(\boldsymbol{c}, \boldsymbol{b}, \boldsymbol{a})$; if these are conjugate, we call the net *symmetric*, and if we adjoin a conjugation that takes $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ to a braid of itself we get the *net reflection group*.

(5) The *net weight* of the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is the triple product $(A-2, B-2, C-2)$ in the Griess–Conway algebra. It can be shown that this has the same value for every triple appearing in the net.

(6) From the net $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ we can obtain other nets by replacing $\boldsymbol{b}$ by another element of the dihedral group $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$. These nets, whose net groups will in general be subgroups of that for $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$, are called its *ancestors*. If $\boldsymbol{ab}$ has order 5, then each of the nets $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ and $(\boldsymbol{a}, \boldsymbol{a}^{\boldsymbol{b}}, \boldsymbol{c})$ is the ancestor of the other; we call them *mates*.

(7) The *modular group* of $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ is the subgroup of $\Gamma = \langle s, t \rangle / \langle z \rangle$ that fixes the triple $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ up to conjugation by a power of $\boldsymbol{abc}$. All triples in a net have the same modular group up to conjugation in Γ.

4. Nets of order up to 7

In this section we describe the notational conventions required to understand Table 1, which lists all nets of order up to 7; we also give some supplementary properties here.

Column 1

This shows the class of the net.

Column 2

This indexes the nets of a given class, where we do not distinguish between a net and its reflection.

Columns 3 and 8

Column 8 shows the net group and Column 3 shows its centralizer in the Monster. Curly brackets indicate an unspecified group of a given order. The net rotation group is appended on the right in square brackets. A star indicates that the net is not conjugate to its reflection. Note that in starred cases we have not tried to achieve consistency in correlating the cyclic order of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ (Columns 5–7) with the orientation of the net as described in Column 9. A dollar sign shows that some rotations that preserve the structure of the net as described in Column 9 are not realized in the Monster (and may or may not be realized in the automorphism group of the net group).

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Column 4

This shows the net weight. Given the class of $\boldsymbol{ab}$, one can compute it as follows. Here V_0 denotes the sum of the axis vectors corresponding to the elements of the outer half of the dihedral group $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$, and V_i $(i > 0)$ denotes the axis vector corresponding to $(\boldsymbol{ab})^i$, where this exists.

1A: $60(V_0, C) - 488$.

2A: $(6V_0 - 8V_1, C) - 40$.

2B: $24 - 2(V_0, C)$.

3A: $(2V_0 - 3V_1, C) - 2$.

3C: $16 - (V_0, C)$.

4A: $(V_0 - V_1, C) + 8$.

4B: $(V_2 - V_0, C) + 16$.

5A: the sum of the weights of $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ and its mate $(\boldsymbol{a}, \boldsymbol{a}^{\boldsymbol{b}}, \boldsymbol{c})$ is $24 - (V_0, C)$.

6A: $14 + (V_2 + V_3 - V_0, C)$.

Columns 5–7

These show Monster elements $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ in the required configuration. Sometimes we use the standard 'projective plane' notation, defined in page 232 of the Atlas [**3**], in which the Bimonster is generated by the standard Y_{555} elements $\{a, b_i, c_i, d_i, e_i, f_i\}$ to which are adjoined redundant generators $\{a_i, f, g_i, z_i\}$ $(1 \leqslant i \leqslant 3)$. In this notation $|PL|$ denotes the diagram automorphism of order 2 that fixes all the lines through the point P and all the points on the line L. $|b_3e_3|$, which we shall also identify with $1T.2E.3X.49.58.67$, interchanges the subscripts 1 and 2, and similarly for the other $|b_ie_i|$.

We also use an alternative notation for some of these elements. The A_{11}-diagram generated by those Y_{555} nodes not having subscript 3 is equated to the symmetric group on points 1–12, where we abbreviate 10, 11 and 12 to X, E and T. Nodes interchange adjacent numbers from $f_1 = (12)$ to $f_2 = (ET)$. (Note: for $L_2(11)$ and related groups, we choose a numbering whereby 1–X correspond to themselves, E corresponds to 0 and T corresponds to ∞.) The diagram is extended to $Y_{551} = O_{10}^-(2) \cdot 2$ by b_3, which acts as the bifid map $(123456|789XET)$, which we may abbreviate to $(123456|-)$; we then abbreviate $(12)(34)(9X)(ET).(123456|-).(123478|-)$, or $12.34.9X.ET.(123456|-).(123478|-)$, to $12.34|56.78|9X.ET$, as it is invariant under permutations of the three 'blocks'. For other elements of $Y_{552} = 3 \cdot Fi_{24}$ we use 'reflection group' notation: if we take the 13-dimensional space consisting of all vectors $(abcdefghijkl|mn)$ where the sum of the coordinates before the (vertical) slash is 6 times the sum of the coordinates after, and use as norm form the sum of the squares of all 14 coordinates minus $5(m+n)^2$, then we can map the isometries that interchange a with b, b with c, …, k with l (i.e. the

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reflections in $(1\bar{1}0000000000|00)$, etc.) to f_1, e_1, ..., b_1, a, b_2, ..., f_2; the reflection in $(000000111111|01)$ to b_3; and the reflection in $(000000000000|1\bar{1})$ to c_3; then any isometry of the 13-space generated by the above reflections, e.g. a reflection in a vector v obtained by reflecting the above vectors in one another, maps to an element of Y_{552}, so we can use v to denote this element.

The other notation we shall use is as follows. Where the notation does not define a unique transposition, either it will not matter which one we use, or it can be seen that one of them does satisfy the required conditions and we assume that is the one we are using.

(1) The $3 \cdot S_7$ centralizing a 7A-type D_{14} is mapped to the S_7 acting on the letters A–G.

(2) The extra-special group $Z_1 = 2^{1+24}$, with centre π, is mapped to $\Lambda/2\Lambda$, where Λ is the Leech lattice. We denote the elements of $\Lambda/2\Lambda$ by corresponding Leech lattice vectors. Let Z_2 be the extra-special group whose centre is an inverse image of $(8, 0^{23})$. We may extend our notation to the group $2^{2+11+22} \cdot 2 = 2^{1+24} \cdot 2^{11}$ generated by Z_1 and $C_{Z_2}(\pi)$. We restrict the map $Z_1 \mapsto \Lambda/2\Lambda$ to $Z_1 \cap Z_2$ and further to a group of order 2^{11} corresponding to the space generated by vectors of type $(4, -4, 0^{22})$. We now re-extend it to a map from Z_2 to $\Lambda/2\Lambda$ and denote an inverse image of the vector $(2^8, 0^{16})$ by a sign change symbol $(-^8, +^{16})$. If we define ω to be an element of order 3 commuting with the 2^{11} group referred to above and inverted by a Z_1-preimage of $(-3, 1^{23})$, we obtain the group $2^{2+11+22} \cdot S_3$. Note that the ordering of the coordinates is not intended to be significant: it should be regarded as arbitrary subject to the displayed vectors being in the Leech lattice.

(3) Elements of Z_1 may also be expressed in 'quaternionic' notation, with coordinates (in the Miracle Octad Generator (MOG) ordering; see the ATLAS [**3**, page 94]) being grouped into six blocks of four. When halved, this will, in order, become the 1, i, j and k coordinates of a quaternion. This notation may be extended to a normalizing group $S_3 \times 3S_6$ acting on the quaternions $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ by permutation, pre- and post-multiplication. We write $\omega = \frac{1}{2}(-1 + i + j + k)$, we write $\bar{\alpha}$ for the conjugate of α (i.e. negating the i, j and k coordinates), and ij, say, denotes the anti-automorphism interchanging the i and j coefficients.

(4) The $2 \cdot M_{22} \cdot 2$ centralizing a suitable $2 \times A_5$ that contains $\langle \pi, \omega \rangle$ is mapped to the subgroup of M_{24} acting on the MOG array, with positions labelled C–Z in order (reading down the columns from left to right), that fixes $\{C, D\}$ (or $\{C, K\}$). We may extend to the $2^{11} \cdot M_{22} \cdot 2$ centralizing $\langle \pi, \omega, (4^2, 0^{22}) \rangle$ by defining, for example, $\overline{EF}$ as EF times a vector of type $(4, -4, 0^{22})$ whose support is $\{E, F\}$. Similarly, we may map the $2^2 \cdot M_{21} \cdot S_3$ normalizing the product of a 7A- and a 2A-pure four-group to the subgroup of M_{24} fixing the set $\{C, D, E\}$. To avoid confusion, if we are in $2 \cdot M_{21} \cdot 2$ as opposed to $2^2 \cdot M_{21} \cdot S_3$, we choose elements so that at least one of $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ takes E to one of F–Z.

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(5) The $U_3(5)$ centralizing a 5A-pure 5^2 is shown by the natural representation of its triple cover by 3×3 matrices over $GF(25)$. ω denotes a cube root of unity, $\theta = \sqrt{-3} = 2\omega + 1$, and a bar over a character denotes the Galois automorphism of the field.

(6) The element $fm2m$ conjugates the $2^2 \cdot {}^2E_6(2)$ centralizing $\langle f_1, f_2, f_3 \rangle$ to the one centralizing $\langle d_3 f_3, (d_3 f_3)^{e_3}, d_3 \rangle$. If the mapping is fully specified, then $fm2m$ will be uniquely defined up to multiplication by a central element. Full details of its action are in the author's GAP programs, which yield 27×27 matrices over $GF(4)$ for generators of $3 \cdot {}^2E_6(2)$ in the above two systems. (Incidentally, the name $fm2m$ comes from the names of the files containing these programs: e62.fmats and e62.mats.)

(7) Elements of the $O_8^+(3)$ generated by the Y_{522} in the Y_{555} diagram that contains f_1 are expressed as products of reflections in eight-dimensional space over $GF(3) = \{0, 1, \bar{1}\}$. The A_8 acting on $\{1, 2, \ldots, 8\}$ as defined, a subset of the $\{1, 2, \ldots, 8, 9, X, E, T\}$ defined above, permutes the eight coordinates in the corresponding order, while c_2, b_3 and c_3 are the reflections in $(1111111\bar{1})$, (00000011) and (11111111), respectively. Reflection group notation may also be used to express, modulo the elementary abelian part, elements of the non-split groups $2^6 \cdot O_5(3) = 2^6 \cdot U_4(2)$ (centralizing a $2^4 \cdot D_{10}$) and $3^8 \cdot O_8^-(3)$. For the latter case we shall separate the last coordinate from the rest by a slash because the norm form is 'Lorentzian'.

(8) The 'dualizer' du is a Bimonster element commuting with $c_i d_i e_i f_i$ and taking b_i to $b_i e_i e_i^{d_i f_i}$, thus dualizing the three S_6s they generate, and whose product with a has order 5.

Column 8

See Column 3.

Column 9

This shows the structure of the net in terms of its faces. For netballs, the faces are denoted by lowercase letters from a as far as necessary. For each face the index letter is followed by its class in Atlas notation (with a prime if the face collapses) and a colon, then the index letters of the surrounding faces in clockwise order; any collapsed vertices or edges are shown by uppercase letters S and T, respectively, in the appropriate position. (We use S' or T' instead if the order of the net is divisible by 3 or 2, respectively; this implies that the net is unfaithful as the modulus cannot be so divisible.) For the first face in each orbit under the net reflection group, this is then followed by the class of $(\boldsymbol{ab})^2\boldsymbol{c}$ (if the face has order at least 4) and $(\boldsymbol{ab})^3\boldsymbol{c}$ (if the face has order 6), in each case followed by the index numbers of the relevant ancestral nets (or question marks if no index numbers have yet been assigned). Orbits under the net reflection group are separated by slashes and are ordered numerically except that a 1-orbit precedes a 2-orbit. For honeycombs this notation is cumbersome and we therefore abbreviate it as follows.

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Let t_1 and t_ω be translations of the honeycomb geometry (not necessarily in the net rotation group) 'across' sides of the hexagon 120° apart. The notation is intended to suggest a homomorphism from the additive group $\langle 1, \omega \rangle$, where ω is a cube root of unity, to $\langle t_1, t_\omega \rangle$. The kernel of this homomorphism is a lattice $\langle n_1, n_2 + \omega n_3 \rangle$ (n_i integers) and this lattice defines the geometric structure of the honeycomb. Our symbol for the honeycomb then consists of $\mathrm{hc}(n_1, n_2+\omega n_3)$ followed by the symbol in the above notation, except that the letters refer not to faces but to *orbits* on faces of the net rotation group (or, if any ambiguous cases turn up, a proper subgroup thereof).

Supplementary information

Below we list all unfaithful nets of order up to 7. They are grouped by the order of the intersection of $\langle \boldsymbol{abc} \rangle$ with the centre of the net group. The modulus of the net is obtained by dividing its order by this number.

- Order 2: 2A1–3, 2A10–11, 2B1–3, 4A1–3, 4A6–8, 4A21, 4B1, 4B4–7, 4B23–24, 4C2–4, 4C9–11, 4D1, 6A1–5, 6A13–14, 6A16–18, 6A23, 6A34, 6A36, 6C1–2, 6C4–5, 6C8, 6C17, 6C19, 6F1–4.
- Order 3: 6A12, 6D1–3, 6D5, 6E2.
- Order 4: 4A17–18.

Finally, only the following nets of order up to 7 are large:

2B7, 3B1, 3C3, 4A17–18, 4A21, 4B21–24, 4C7–11, 4D1, 5B1, 6A34, 6A36, 6C16–17, 6C19, 6D5, 6D8, 6E2–7, 6F6, 7B2–3.

How were all these results obtained? The following outlines the general procedure for classifying nets of a given class X.

(1) Use structure constant theory to calculate the 'mass' of triples $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$, where $\boldsymbol{ab}$ belongs to a given class, and also the mass of collapsed vertices, edges and faces. See [**10**, **11**] for further details.

(2) Guess a possible group G (as large as possible) that can centralize the net group of $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$. Then $C_{\mathbb{M}}(G)$ will be the net group or an overgroup thereof.

(3) Confirm the existence of such a net by structure constant calculations within $C_{\mathbb{M}}(G)$.

(4) Work within $C_{\mathbb{M}}(G)$ to find an explicit triple that satisfies the relevant properties, and braid this triple to obtain the net they lie in.

(5) Remove all triples in this net from all relevant structure constant calculations, and repeat the process until the entire mass of triples whose product lies in X is accounted for.

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5. Nets of higher order

When the order of a net gets above 7 the complete enumeration of the previous section becomes more difficult. The net groups get bigger and harder to work in, and for orders 8 and 9 their structures are liable to involve complicated 2-groups and 3-groups, respectively. However, there is one easy case, namely 13B. This has two netballs that each have a triangle, a square, a hexagon, a collapsed vertex and a collapsed edge, with net group $L_3(3)$ that centralizes $13 \cdot 6$ in $\mathbb{M}$; and there are four honeycombs such that $\langle \boldsymbol{ab}, \boldsymbol{c} \rangle = 6 \cdot Suz$, in which $\boldsymbol{ab}, \boldsymbol{c}$ and $\boldsymbol{abc}$ belong to the respective $6 \cdot Suz$-classes −6B (or −6C), −2A and 13A (or 13B). (We obtain the class −2A, say, by taking the $6 \cdot Suz$-class in the lift of 2A whose character values are shown explicitly in the character table of $6 \cdot Suz$ given in the Atlas, and multiplying by the central involution −1.) If one decomposes $\boldsymbol{ab}$ as the product of transpositions $\boldsymbol{a}$ and $\boldsymbol{b}$, then the net group $\langle \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \rangle$ will be the entire Monster.

However, though the complete *enumeration* of nets for most higher orders is harder, in many cases one can *count* them. This is done by using character theory, as described in [**10**, **11**], and requires that one can write down a list of possible net groups and that one has all their character tables. This is likely to happen when the net order is divisible by a large prime (at least 17).

The counting is made easier if the element is self-centralizing (so that the net rotation group is trivial) and if all faces have class 3C, 4B, 5A or 6A (as elements of these classes extend uniquely to a dihedral group in which the outer elements are all transpositions). We conclude this paper by showing the results for a selection of classes, all of which turn out to be perfect.

For each net class covered, and each possible net group, we first give the total number of collapsed vertices (shown as S), collapsed edges (shown as T) and faces of various classes. For collapsed faces the collapsed order is shown as a subscript, and for faces of class 6A that collapse to 2-gons a superscript '$a + b$' means that there are a faces with exponent $2m/3$ and b faces with exponent $m/3$, where m is the modulus. We then show the total defect, the total number of flags, the total exponent and the number of nets with the distribution of flags between them (where this is determined). Thus the case 119A has 3 collapsed vertices, 2 collapsed edges, 3, 2, 29 and 211 faces of respective class 3C, 4B, 5A and 6A, total defect 60, 1428 flags, total exponent 595, and 5 nets, of which 2 have 357 flags and 3 have 238 flags. The notation '119AB' means that the classes 119A and 119B are inverses, so there are another 5 nets of class 119B that can be obtained by reflecting the nets of class 119A. The total number of nets counted below is 731.

- Net class 41A, net group $\mathbb{M}$, $S^9T^{71}3\mathrm{C}^7 4\mathrm{B}^{71} 5\mathrm{A}^{1724} 6\mathrm{A}^{4109}$, defect 2136, 33 579 flags, exponent 7298, 178 nets, $246^{98}123^{71}82^9$.

- Net class 51A, net group $3 \cdot Fi_{24}'$, $T^6 3\mathrm{C}^{12} 4\mathrm{B}^6 5\mathrm{A}^{42} 6\mathrm{A}^{57}$, defect 108, 612 flags, exponent 153, 9 nets, $102^3 51^6$.

- Net class 51A, net group $\mathbb{M}$, $T^{31} 3\mathrm{C}^{21} 4\mathrm{B}^{31} 5\mathrm{A}^{678} 6\mathrm{A}_2^{24+7} 6\mathrm{A}^{2003}$, defect 1020, 15 657 flags, exponent 4335, 85 nets, composition not determined.

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- Net class 57A, net group $\mathbb{M}$, $T^{12}3C^{18}4B^{12}5A^{66}6A^{441}$, defect 180, 3078 flags, exponent 855, 15 nets, 342^3171^{12}.
- Net class 59AB, net group $\mathbb{M}$, $S^5T^{25}3C^54B^{25}5A^{320}6A^{1140}$, defect 480, 8555 flags, exponent 2360, 40 nets, $354^{10}177^{25}118^5$.
- Net class 62AB, net group $2 \cdot B$, $S^1T^64B^{30}5A^{192}6A^{281}$, defect 192, 2356 flags, exponent 496, 16 nets, composition not determined.
- Net class 62AB, net group $\mathbb{M}$, $S^13C^14B^{30}5A^{335}6A_3^{30}6A^{1256}$, defect 492, 9424 flags, exponent 2542, 41 nets, $372^{10}186^{30}124^1$.
- Net class 68A, net group $2 \cdot B$. If one quotients out the central involution of the net group, then either exactly one of the transpositions $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ projects to an involution of B-class 2A, or they all project to B-class 2B. The respective net enumerations are $4B_2^24B^65A^46A^{26}$, defect 24, 204 flags, exponent 68, 2 nets, 102^2, and $S^44B_2^34B^{14}5A^{28}6A^{108}$, defect 84, 850 flags, exponent 238, 7 nets, composition not determined.
- Net class 68A, net group $\mathbb{M}$, $4B^{30}5A^{102}6A_3^66A^{108}$, defect 180, 4896 flags, exponent 1020, 15 nets, 408^9204^6.
- Net class 69AB, net group $3 \cdot Fi_{24}'$, $T^43C^64B^45A^{22}6A^{45}$, defect 60, 414 flags, exponent 115, 5 nets, 138^169^4.
- Net class 69AB, net group $\mathbb{M}$, $T^{20}3C^64B^{20}5A^{218}6A_2^{8+4}6A^{854}$, defect 384, 6348 flags, exponent 2208, 32 nets, composition not determined.
- Net class 71AB, net group $\mathbb{M}$, $S^4T^{14}3C^44B^{14}5A^{169}6A^{724}$, defect 624, 5254 flags, exponent 1562, 22 nets, $426^4213^{14}142^4$.
- Net class 87AB, net group $3 \cdot Fi_{24}'$, $T^53C^54B^55A^{20}6A^{210}$, defect 60, 435 flags, exponent 145, 5 nets, 87^5.
- Net class 87AB, net group $\mathbb{M}$, $T^63C^54B^65A^{67}6A_2^56A^{342}$, defect 132, 2436 flags, exponent 812, 11 nets, 261^6174^5.
- Net class 93AB, net group $\mathbb{M}$, $T^53C^54B^55A^{20}6A^{210}$, defect 60, 1395 flags, exponent 465, 5 nets, 279^5.
- Net class 94AB, net group $2 \cdot B$, $S^2T^54B^{12}5A^{37}6A^{110}$, defect 84, 893 flags, exponent 329, 7 nets, 141^594^2.
- Net class 94AB, net group $\mathbb{M}$, $4B^85A^{56}6A_3^86A^{320}$, defect 96, 2256 flags, exponent 752, 8 nets, 282^8.
- Net class 95AB, net group $\mathbb{M}$, $S^1T^{12}3C^14B^{12}5A^{74}5A_1^36A^{417}$, defect 156, 2926 flags, exponent 1235, 13 nets, $285^9190^157^3$.
- Net class 119AB, net group $\mathbb{M}$, $S^3T^23C^34B^25A^{29}6A^{211}$, defect 60, 1428 flags, exponent 595, 5 nets, 357^2238^3.

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Table 1.

$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
1A	1	$2^2.{}^2E_6(2)[.S_3]$	-1704	12.34	13.24	14.23	2^2	a2A : bc; b2A : ca; c2A : ab
2A	1	$2.B$	14872	12.34	12.34	12.34	2	a1A : aST'
2A	2	$2^2.{}^2E_6(2)$	1432	12.34	12.34	13.24	2^2	a1A : b; b2A : abT'
2A	3	$2^{2+22}.Co_2$	-488	12.34	12.34	56.78	2^2	a1A : b; b2B : abT'
2A	4	Fi_{23}	292	12.34	12.34	12.45	S_3	a1A : b; b3A : $abSb$
2A	5	Th	-248	$\lvert b_1e_1\rvert$	$\lvert b_1e_1\rvert$	$\lvert b_2e_2\rvert$	S_3	a1A : b; b3C : $abSb$
2A	6	$2^{1+22}.McL$	-8	12.34	12.34	$\lvert b_3e_3\rvert$	D_8	a1A : c; b2B$'$: c; c4A : $acbc$(2A3$\vert$6)
2A	7	$2.F_4(2)$	-248	12.34	12.34	15.26	D_8	a1A : c; b2A$'$: c; c4B : $acbc$(2A2$\vert$7)
2A	8	$HN[.2]$	-128	14.23	14.23	25.34	D_{10}	a1A : c; b1A : d; c5A : $dcacd$(2A8); d5A : $cdbdc$
2A	9	$2.Fi_{22}$	-188	12.34	12.34	45.67	$2\times S_3$	a1A : d; b2A : dc; c3A : dbd; d6A : $dadcbc$(2A4$\vert$9, 2A2$\vert 9^2$)
2A	10	$2^{3+20}.U_6(2)[.2]$	-104	12.34	12.56	56.78	2^3	a2B : bc; b2A : ca; c2A : ab
2A	11	$2^{3+10+20}.M_{22}.2[.S_3]$	24	12.34	56.78	$9X.ET$	2^3	a2B : bc; b2B : ca; c2B : ab
2A	12	$O_8^+(3)[.A_4]$	-194	12.45	12.56	23.56	$3^2.2$	a3A : bcd; b3A : adc; c3A : dab; d3A : cba
2A	13	$2^{2+20}.U_4(3)[.2^2]$	-200	12.34	$12.9X$	$\lvert b_3e_3\rvert$	$2\times D_8$	a2A : cd; b2A : cd; c4A : $adbd$(2A6$\vert$10); d4A : $acbc$
2B	1	$2^{3+20}.U_6(2)[.S_3]$	344	12.34	12.56	12.78	2^3	a2A : bc; b2A : ca; c2A : ab
2B	2	$2^{3+7+16}.S_6(2)[.2]$	-40	12.34	12.56	$78.9X$	2^3	a2A : bc; b2B : ca; c2B : ab
2B	3	$2^{3+4+12+8}.A_8[.S_3]$	24	b_1d_1	b_2d_2	b_3d_3	2^3	a2B : bc; b2B : ca; c2B : ab
2B	4	$2^{1+21}.M_{22}[.2^2]$	56	$4^2, 0^{22}$	$-3, 1^{23}$	$1, -3, 1^{22}$	$2\times D_8$	a2A : cd; b2A : cd; c4A : $adbd$(2A6$\vert$9); d4A : $acbc$
2B	5	$2^{1+21}.2^4.A_7[.2^2]$	-8	$-3, 1^{23}$	$-2^8, 0^{16}$	$3, -1^7, 1^{16}$	$2\times D_8$	a2B : cd; b2B : cd; c4A : $adbd$(2A6$\vert$10); d4A : $acbc$
2B	6	$2^{2+14}.S_6(2)[.2]$	8	12.34	12.56	17.28	$2\times D_8$	a2A : cd; b2B : cd; c4B : $adbd$(2A7$\vert$9); d4B : $acbc$
2B	7	$2^2.U_6(2)[.S_3]$	20	12.45	12.67	13.89	$2^2\times S_3$	a2A : ef; b2A : fd; c2A : de; d6A : $ecefbf$(2A9^2, 2B1$\vert 7^2$); e6A : $fafdcd$; f6A : $dbdeae$
3A	1	$2.Fi_{22}[.2]$	78	12.34	12.56	34.67	$2\times S_3$	a2A : bc; b2A : ac; c6A$'$: ab(6A1^2, 1A1^3)
3A	2	$S_8(2)$	-42	12.34	12.56	23.56	S_4	a2A : bc; b3A : cac; c4B : $abcTb$(4B1$\vert$2)

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
3A	3	$A_{12}[.2]$	−72	12.34	13.24	23.45	A_5	a2A:de; b3A:ecd; c3A:dbe; d5A:$ceaeb$(5A1); e5A:$bdadc$
3A	4	$O_7(3)[.S_3]$	−84	12.45	23.78	56.78	$S_3\times S_3$	a2A:de; b2A:de; c2A:de; d6A:$aebece$($6\mathrm{A}1\mid 9, 3\mathrm{A}1^3$); e6A:$cdbdad$
3A	5	$2^{11}.M_{22}[.2]$	−66	12.78	17.29.38. $4X.5E.6T$	18.27.3X. 49.5E.6T	$2\times S_4$	a2A:ce; b2A:de; c4A:$aede$($4\mathrm{A}3\mid 5$); d4A:$bece$; e6A:$cacdbd$($6\mathrm{A}4^2, 2\mathrm{B}1\mid 4^2$)
3A	6	$2^{6+8}.A_7[.S_3]$	−74	12.34	17.28.39. $4X.5E.6T$	18.29.37. $4X.5E.6T$	$2^4.S_3$	a3A:dec; b3A:ced; c4A:$aebd$($4\mathrm{A}4\mid 8$); d4A:$beac$; e4A:$adbc$
3B	1	$3^5.O_5(3)[.3^2.6]$	−3	12.56	23.78	45.89	$3^3.2^2$	hc$(3,3\omega)$: a^9:a^6($6\mathrm{A}9^2, 3\mathrm{A}4^3$)
3C	1	${}^3D_4(2).3$	−24	b_1b_2	$\lvert b_1e_1\rvert$	$\lvert b_2e_2\rvert$	S_4	a2A:bc; b3C:cac; c4B:$abcTb$($4\mathrm{B}1\mid 3$)
3C	2	$U_3(8).3[.2]$	0	$AB.CD$	$CD.EF$	$BC.DE$	A_5	a2A:de; b3C:ecd; c3C:dbe; d5A:$ceaeb$(5A2); e5A:$bdadc$
3C	3	$G_2(3)[.S_3]$	−12	AB	DE	$BC.EF$	$3^{1+2}.2^2$	a2A:fg; b2A:hi; c2A:jk; d6A:$fghijk$($6\mathrm{A}9^2, 3\mathrm{C}3^3$); e6A:$kjihgf$; f6A:$gagdke$($6\mathrm{A}9\mid 10, 3\mathrm{A}1\mid 4^2$); g6A:$fafehd$; h6A:$ibidge$; i6A:$hbhejd$; j6A:$kckdie$; k6A:$jcjefd$
4A	1	$2^{1+21}.M_{21}.2[.2]$	8	$4^2, 0^{22}$	$4, -4, 0^{22}$	$1^2, -3, 1^{21}$	$2\times D_8$	a2B:bc; b2B:ac; c4A′:ab($2\mathrm{A}3^2$)
4A	2	$2^{1+21}.U_4(3)[.2]$	136	$1, -3, 1^{22}$	$4, -4, 0^{22}$	$0, 4^2, 0^{21}$	$2\times D_8$	a2A:bc; b2A:ac; c4A′:ab($2\mathrm{A}2^2$)
4A	3	$2^{1+21}.M_{22}$	−56	$4^2, 0^{22}$	$4, -4, 0^{22}$	$-3, 1^{23}$	$2\times D_8$	a2A:bc; b2B:ac; c4A′:ab($2\mathrm{A}2\mid 3$)
4A	4	$2^{11}.M_{23}$	−28	12.56	23.67	34.78	S_4	a2B′:b; b3A:$abSb$
4A	5	$2^{11}.M_{22}$	4	23.XT	34.9X	$\lvert b_3e_3\rvert$	$2\times S_4$	a2B′:d; b2A:dc; c3A:dbd; d6A:$dadcbc$($4\mathrm{A}4\mid 5, 2\mathrm{A}10\mid 9^2$)
4A	6	$2^{1+15}.U_4(2)[.2^2]$	−56	1T.23	1T.45	$\lvert b_3e_3\rvert$	$2^{1+3}.2$	a2A:cd; b2A:cd; c4B:$adbd$($2\mathrm{B}1\mid 6$); d4B:$acbc$
4A	7	$2^{1+15}.A_8[.2^2]$	8	12.34	56.78	13.57	$2^{1+3}.2$	a2B:cd; b2B:cd; c4B:$adbd$($2\mathrm{B}2\mid 6$); d4B:$acbc$
4A	8	$2^{2+10+8}.A_7[.2]$	−8	$-^8, +^{16}$	$2^8, 0^{16}$	$0^7, 4^2, 0^{15}$	$2^{1+3}.2$	a2A:cd; b2B:cd; c4A:$adbd$($2\mathrm{B}2\mid 5$); d4A:$acbc$

$\boldsymbol{abc}$	#	$C_M\langle\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
4A	9	$2^{1+14}.A_7$	-32	$-^8, +^{16}$	$0^7, 4^2, 0^{15}$	$1^8, -3, 1^{15}$	$2^{1+4}.2$	$a2\text{A}: cd$; $b4\text{A}': cd(2\text{B}5\mid 6)$; $c4\text{A}: adbd(4\text{A}3\mid 7)$; $d4\text{B}: acbc(4\text{A}2\mid 8)$
4A	10	$2^{11}.M_{21}[.S_3]$	-44	$16.25.37.$ $48.9X.ET$	$15.27.36.$ $48.9E.XT$	$15.26.38.$ $47.9T.XE$	$2^2 \times S_4$	$a2\text{A}: ef$; $b2\text{A}: fd$; $c2\text{A}: de$; $d6\text{A}: fbfece(4\text{A}5^2, 2\text{B}1\mid 7^2)$; $e6\text{A}: dcdfaf$; $f6\text{A}: eaedbd$
4A	11	$2^6.U_4(2)[.2^2]$	-32	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$17.23.89.$ $4E.5X.6T$	$2^4.D_{10}$	$a2\text{A}: cd$; $b2\text{A}: ef$; $c5\text{A}: fdade(4\text{B}15)$; $d5\text{A}: ecacf$; $e5\text{A}: dfbfc$; $f5\text{A}: cebed$
4A	12	$2.M_{21}.2[.2]$	-40	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$17.23.89.$ $4X.5T.6E$	$2 \times A_6$	$a3\text{A}: ecf$; $b3\text{A}: fde$; $c4\text{A}: faed(6\text{A}5\mid 23)$; $d4\text{A}: ebfc$; $e5\text{A}: cafbd(10\text{A}?)$; $f5\text{A}: dbeac$
4A	13	$2^{6+4}.A_7[.2]$	-44	$12.XE$	$23.9X$	$\lvert b_3e_3\rvert$	$2^4.D_{12}$	$a2\text{A}: ef$; $b3\text{A}: fde$; $c3\text{A}: edf$; $d4\text{A}: ebfc(4\text{A}4\mid 5)$; $e6\text{A}: fbdcfa(4\text{B}2\mid 18, 4\text{A}2\mid 5^2)$; $f6\text{A}: ecdbea$
4A	14	$2^6.A_8[.2^2]$	-34	12.56	23.67	34.58	$\frac{1}{2}(S_4 \times S_4)$	$a3\text{A}: ebf$; $b3\text{A}: fae$; $c3\text{A}: fde$; $d3\text{A}: ecf$; $e6\text{A}: bafcdf(4\text{B}13^2, 4\text{B}12\mid 2^2)$; $f6\text{A}: abedce$
4A	15	$2.M_{22}$	-38	12.78	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$2 \times S_5$	$a2\text{A}: ef$; $b3\text{A}: cfd$; $c4\text{A}: defb(10\text{A}?\mid ?)$; $d4\text{A}: cbfe$; $e5\text{A}: dfafc(6\text{A}22)$; $f6\text{A}: aedbce(6\text{A}4\mid 23, 6\text{A}1\mid 22^2)$
4A	16	$2.U_4(3)[.2]$	-26	13.45	$23.XE$	$\lvert b_3e_3\rvert$	$2 \times 3^2.D_8$	$a2\text{A}: cf$; $b2\text{A}: de$; $c4\text{A}: afef(6\text{A}3\mid 13)$; $d4\text{A}: befe$; $e6\text{A}: fdbdfc(6\text{A}7\mid 9, 4\text{A}2\mid 16^2)$; $f6\text{A}: ecaced$
4A	17	$2^{1+21}.U_3(5)[.S_4]$	-88	$0^7, 4^2, 0^{15}$	$2^8, 0^{16}$	$1^7, -3, 1^{16}$	4.2^2	$a4\text{A}: cedf(2\text{A}6^2)$; $b4\text{A}: fdec$; $c4\text{A}: eafb$; $d4\text{A}: bfae$; $e4\text{A}: acbd$; $f4\text{A}: dbcasb$
4A	18	$2^{1+21}.3^4.M_{10}[.S_4]$	40	$0, 4^2, 0^{21}$	$-3, 1^{23}$	$-3, 1^{11}, -1^{12}$	4.2^2	$a4\text{A}: cedf(2\text{A}6^2)$; $b4\text{A}: fdec$; $c4\text{A}: eafb$; $d4\text{A}: bfae$; $e4\text{A}: acbd$; $f4\text{A}: dbca$
4A	19	$U_3(5)[.S_4, *]$	-38	14.23	$1X.29.38.$ $47.56.ET$	$1X.28.36.$ $49.57.ET$	$5^2.4.2^2$	$a4\text{A}: cedf(10\text{A}?^2)$; $b4\text{A}: fdec$; $c4\text{A}: eafb$; $d4\text{A}: bfae$; $e4\text{A}: acbd$; $f4\text{A}: dbca$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
4A	20	$3^4.A_6[.S_4]$	−41	$12.XE$	$14.25.36.$ $7X.8E.9T$	$19.28.37.$ $4X.5E.6T$	$3^4.4.2^2$	a4A : $cedf$(6A7^2); b4A : $fdec$; c4A : $eafb$; d4A : $bfae$; e4A : $acbd$; f4A : $dbca$
4A	21	$2^{3+6+6+6}.3^2.2[.S_4]$	−24	$-^8,+^{16}$	$0^4,2^8,0^{12}$	$4,0^7,4,0^{15}$	$2^3.2^3$	a4A : $cedf$(2B5^2); b4A : $fdec$; c4A : $eafb$; d4A : $bfae$; e4A : $acbd$; f4A : $dbca$
4A	22	$2^{2+8+6}.3^2.2[.S_4]$	−48	$-^3,+^{16},-^5$	$2^4,0^{12},2^4,0^4$	$-^7,+^{16},-\times$ $2^7,0^{16},2$	$2^2.2^3.2^3$	a4A : $cedf$(4A8^2); b4A : $fdec$; c4A : $eafb$; d4A : $bfae$; e4A : $acbd$; f4A : $dbca$
4B	1	$2.F_4(2)[.2]$	144	12.34	12.56	13.24	D_8	a2A : bc; b2A : ac; c4B$'$: ab(1A1^2)
4B	2	$S_8(2)$	84	12.56	23.56	34.56	S_4	a2A$'$: b; b3A : $abSb$
4B	3	${}^3D_4(2).3$	−24	$AB.CD$	$AC.BD$	$BE.CF$	S_4	a2A$'$: b; b3C : $abSb$
4B	4	$2^{1+15}.2^5.S_6[.2]$	16	12.34	13.56	$78.9X$	$2\times D_8$	a2B : bc; b2B : ac; c4B$'$: ab(2B2^2)
4B	5	$2^{1+15}.S_6(2)$	−48	12.34	56.78	13.56	$2\times D_8$	a2A : bc; b2B : ac; c4B$'$: ab(2B1|2)
4B	6	$2^{1+15}.U_4(2)$	0	$-^8,+^{16}$	$0^6,2^8,0^{10}$	$0^5,4^2,0^{17}$	$2^{1+3}.2$	a2A : cd; b2B : cd; c4A : $adbd$(2B2|6); d4B : $acbc$(2A10|13)
4B	7	$2^{1+14}.2^3.L_3(2)[.2]$	0	$-^8,+^{16}$	$0^6,2^8,0^{10}$	$0^{13},4^2,0^9$	$2^2.2^2.2$	a2B : cd; b2B : cd; c4A : $adbd$(2B3|6); d4B : $acbc$(2B3|5)
4B	8	$2^{1+14}.3^3.S_4[.2]$	−24	$-^8,+^{16}$	$0^6,2^8,0^{10}$	$1^{14},-3,1^9$	$2^{1+4}.2$	a2A : cd; b4A$'$: cd(2B6^2); c4B : $adbd$(4A2|6); d4B : $acbc$
4B	9	$2^{1+14}.S_5[.2]$	8	$-^8,+^{16}$	$0^6,2^8,0^{10}$	$1^8,-3,1^{15}$	$2^{1+4}.2$	a2B : cd; b4A$'$: cd(2B6^2); c4B : $adbd$(4A3|6); d4B : $acbc$
4B	10	$2^{1+14}.A_6[.2]$	−8	$-^8,+^{16}$	$0^7,4^2,0^{15}$	$-3,1^{23}$	$2^{1+4}.2$	a2B : cd; b4A$'$: cd(2B5^2); c4A : $adbd$(4A1|8); d4A : $acbc$
4B	11	$2^{1+14}.A_7[.2]$	24	$-^8,+^{16}$	$4,0^{22},-4$	$-3,1^{23}$	$2^{1+4}.2$	a2A : cd; b4A$'$: cd(2B5^2); c4A : $adbd$(4A3|8); d4A : $acbc$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
4B	12	$2^7.S_6(2)$	-12	12.56	23.56	34.78	$2\times S_4$	$a2\mathrm{A}'\colon d;\ b2\mathrm{B}\colon cd;\ c3\mathrm{A}\colon dbd;$ $d6\mathrm{A}\colon dadcbc(4\mathrm{B}2\mid 12, 2\mathrm{A}10\mid 9^2)$
4B	13	$O_8^+(2)[.3]$	30	12.56	23.67	34.67	$\frac{1}{2}(S_3\times S_4)$	$a3\mathrm{A}\colon cdb;\ b3\mathrm{A}\colon adc;\ c3\mathrm{A}\colon bda;$ $d6\mathrm{A}'\colon cba(2\mathrm{A}12^2, 4\mathrm{B}2^3)$
4B	14	$2^{1+10}.S_6[.S_3]$	4	12.56	23.78	$34.9X$	$2^2\times S_4$	$a2\mathrm{B}\colon ef;\ b2\mathrm{B}\colon fd;\ c2\mathrm{B}\colon de;$ $d6\mathrm{A}\colon ecefbf(4\mathrm{B}12^2, 2\mathrm{B}2\mid 7^2);$ $e6\mathrm{A}\colon fafdcd;\ f6\mathrm{A}\colon dbdeae$
4B	15	$2^6.U_4(2)[.2^2]$	0	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$17.29.38.$ $45.XE.6T$	$2^4.D_{10}$	$a2\mathrm{A}\colon cd;\ b2\mathrm{A}\colon ef;\ c5\mathrm{A}\colon fdade(4\mathrm{A}11);$ $d5\mathrm{A}\colon ecacf;\ e5\mathrm{A}\colon dfbfc;\ f5\mathrm{A}\colon cebed$
4B	16	$2^6.U_4(2)[.2^2]$	0	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$17.23.89.$ $45.XE.6T$	$2^4.D_{10}$	$a2\mathrm{B}\colon cd;\ b2\mathrm{B}\colon ef;\ c5\mathrm{A}\colon fdade(4\mathrm{B}16);$ $d5\mathrm{A}\colon ecacf;\ e5\mathrm{A}\colon dfbfc;\ f5\mathrm{A}\colon cebed$
4B	17	$\frac{1}{2}S_6\wr 2[.2]$	0	12.34	23.45	14.56	A_6	$a3\mathrm{A}\colon ecf;\ b3\mathrm{A}\colon fde;\ c4\mathrm{B}\colon faed(3\mathrm{A}2\mid 3);$ $d4\mathrm{B}\colon ebfc;\ e5\mathrm{A}\colon cafbd(5\mathrm{A}4);\ f5\mathrm{A}\colon dbeac$
4B	18	$2^{6+4}.A_7[.2]$	4	12.67	$16.28.37.$ $45.9X.ET$	$15.26.37.$ $48.9X.ET$	$2^4.D_{12}$	$a2\mathrm{B}\colon ef;\ b3\mathrm{A}\colon fde;\ c3\mathrm{A}\colon edf;$ $d4\mathrm{A}\colon ebfc(4\mathrm{A}4\mid 5);$ $e6\mathrm{A}\colon fbdcfa(4\mathrm{A}4\mid 13, 4\mathrm{A}3\mid 5^2);\ f6\mathrm{A}\colon ecdbea$
4B	19	S_{10}	-6	12.34	23.45	45.67	S_5	$a2\mathrm{A}\colon ef;\ b3\mathrm{A}\colon cfd;\ c4\mathrm{B}\colon defb(5\mathrm{A}1\mid 3);$ $d4\mathrm{B}\colon cbfe;\ e5\mathrm{A}\colon dfafc(6\mathrm{A}21);$ $f6\mathrm{A}\colon eaedbc(3\mathrm{A}2\mid 3, 6\mathrm{A}1\mid 21^2)$
4B	20	$L_4(3).2[.2]$	-18	$12.XE$	23.45	$\lvert b_3e_3\rvert$	$3^2.D_8$	$a2\mathrm{A}\colon cf;\ b2\mathrm{A}\colon de;\ c4\mathrm{B}\colon afef(3\mathrm{A}1\mid 4);$ $d4\mathrm{B}\colon befe;\ e6\mathrm{A}\colon dbdfcf(6\mathrm{A}6\mid 9, 4\mathrm{B}1\mid 20^2);$ $f6\mathrm{A}\colon cacede$
4B	21	$2^{3+6}.L_3(2)[.A_4]$	-2	$\lvert b_3e_3\rvert$	$\lvert d_3a_1\rvert$	$17.56\mid 2T.$ $4X\mid 39.8E$	$2^6.3^2.2$	$a3\mathrm{A}\colon gfh;\ b3\mathrm{A}\colon egi;$ $c3\mathrm{A}\colon fej;\ d3\mathrm{A}\colon jih;$ $e6\mathrm{A}\colon jcfgbi(4\mathrm{A}14^2, 4\mathrm{B}2\mid 12^2);$ $f6\mathrm{A}\colon hagecj;\ g6\mathrm{A}\colon ibefah;\ h6\mathrm{A}\colon gafjdi;$ $i6\mathrm{A}\colon ebghdj; j6\mathrm{A}\colon fceidh$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\rangle$	net
4B	22	$2^{6+4}.A_6[.2^2]$	-12	12.67	16.28.37. 45.9$X.ET$	15.26.37. 48.9$E.XT$	$2^4.(2^2 \times S_3)$	a2A : gh; b2A : ij; c4A : $heif$(4A5$\vert$5); d4A : $jegf$; e6A : $ichgdj$(4B18^2, 2B4$\vert$7^2); f6A : $hcijdg$; g6A : $ehahfd$(4A5$\vert$13, 4A2$\vert$10^2); h6A : $fgagec$; i6A : $ejbjfc$; j6A : $fibied$;
4B	23	$2^{1+14}.\frac{1}{2}S_4 \wr 2[.D_8]$	16	$-^8, +^{16}$	$0^7, 4, 0^{15}, 4$	$0^6, 2^8, 0^{10}$	$2^3.2^3$	a4B : $cedf$(2A13^2); b4B : $fdec$; c4A : $eafb$(2B5$\vert$6); d4A : $bfae$; e4A : $acbd$; f4A : $dbca$
4B	24	$2^{1+14}.2^4.5.4[.D_8]$	-16	$-^8, +^{16}$	$4, 0^7, 4, 0^{15}$	$0^6, 2^8, 0^{10}$	$2^3.2^3$	a4B : $cedf$(2B4^2); b4B : $fdec$; c4A : $eafb$(2B5$\vert$6); d4A : $bfae$; e4A : $acbd$ f4A : $dbca$
4B	25	$2^{2+8+4}.3^2.2[.D_8]$	-8	$-, +^{16}, -^7$	$2^4, 0^{12}, 2^4, 0^4$	$-^7, +^{16}, - \times$ $2^7, 0^{16}, 2$	$2^2.2^3.2^3$	a4B : $cedf$(4A8^2); b4B : $fdec$; c4A : $eafb$(4A7$\vert$8); d4A : $bfae$; e4A : $acbd$; f4A : $dbca$
4C	1	$2^5.2^4.A_8$	16	$\vert b_1e_1\vert$	$\vert b_2e_2\vert$	$\vert b_1e_1\vert.d_2d_3f_2f_3$	S_4	a2B$'$: b; b3C : $abSb$
4C	2	$2^{1+14}.(2 \times A_8)[.2]$	8	$-^8, +^{16}$	$0^{15}, 4^2, 0^7$	$0^8, 2^8, 0^8$	$2 \times D_8$	a2B : bc; b2B : ac; c4A$'$: ab(2B3^2)
4C	3	$2^{1+14}.2^4.L_3(2)[.2^2]$	-8	$-^8, +^{16}$	$0^7, 4^2, 0^{15}$	$0^8, 2^8, 0^8$	$2^{1+3}.2$	a2B : cd; b2B : cd; c4A : $adbd$(2B3$\vert$5); d4A : $acbc$
4C	4	$2^{1+11}.2^6.3^2.2^2[.2^2]$	8	$+^5, -^2, +^{11}, -^6$	$2^8, 0^{16}$	$0^4, -2^2, 2^6, 0^{12}$	$2.2^2.2^2$	a2B : cd; b2B : cd; c4B : $adbd$(2B3$\vert$6); d4B : $acbc$
4C	5	$2^{1+14}.L_3(2)$	0	$-^8, +^{16}$	$1^{23}, -3$	$0^7, 4^2, 0^{15}$	$2^{1+4}.2$	a2B : cd; b4A$'$: cd(2B5$\vert$6); c4A : $adbd$(4A1$\vert$7); d4B : $acbc$(4A3$\vert$8)

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
4C	6	$2^{3+2+6}.L_3(2)[.2^2]$	0	$(\alpha,\beta)(\gamma,\delta)$	$(\beta^\omega,\gamma)(\delta^\omega,\epsilon)$	$(\alpha,\beta+2).$ $(\gamma,\delta+2)$	$2^4.D_{10}$	a2B$:cd$; b2B$:ef$; c5A$:dadef$(4C6); d5A$:cacfe$; e5A$:fbfcd$; f5A$:ebedc$
4C	7	$2^7.2^3.L_3(2)[.2^2]$	4	$56.b_3$	$17.24.35.$ $6T.8X.9E$	$13.27.45.$ $6T.8E.9X$	$2^4.(2^2\times S_3)$	a2B$:gh$; b2B$:ij$; c4A$:heif$(4A5|5); d4A$:jegf$; e6A$:ichgdj$(4A13^2, 2B4\|7^2); f6A$:hcijdg$; g6A$:ehahfd$(4B12\|18, 4A3\|10^2); h6A$:fgagec$; i6A$:ejbjfc$; j6A$:fibied$
4C	8	$2^{2+3+6}.S_3[.4^2.6]$	-2	$\|b_3e_3\|$	$1T.46\|2E.$ $7X\|35.89$	$19.27\|3X.$ $8T\|46.5E$	$2^8.3^2.2$	hc$(4,4\omega)$: $a^{16}:a^6$(4B21^2, 4B12^3)
4C	9	$2^{1+14}.U_3(3)[.D_8]$	24	$-^6,+^3,-^2,+^{13}$	$0^9,2^4,0^7,2^4$	$0^6,2^6,-2^2,0^{10}$	$2^{1+3}.2$	a4A$:cedf$(2B6^2); b4A$:fdec$; c4B$:eafb$(2A7|13); d4B$:bfae$; e4B$:acbd$; f4B$:dbca$
4C	10	$2^{1+14}.2^6.S_3[.S_4]$	8	$+^4,-^4,+^{12},-^4$	$2^3,-2^2,2^3,0^{16}$	$0^4,2^8,0^{12}$	$2^3.2^3$	a4A$:cedf$(2B5^2); b4A$:fdec$; c4A$:eafb$; d4A$:bfae$; e4A$:acbd$; f4A$:dbca$
4C	11	$2^{1+14}.2^4.S_3[.D_8]$	-8	$+^4,-^2,+^{12},-^6$	$2^3,-2^2,2^3,0^{16}$	$0^4,2^8,0^{12}$	$2^{1+4}.2$	a4A$:cedf$(2B6^2); b4A$:fdec$; c4B$:afbe$(2B5|6); d4B$:bfae$; e4B$:acbd$; f4B$:dbca$
4C	12	$2^{2+8+6}.S_3[.S_4]$	16	$1^7,-3,1^{16}$	$+^4,-^8,+^{12}$	$-^8,+^{16}\times$ $0^3,4^2,0^{19}$	$2^2.2^3.2^3$	a4A$:cedf$(4A8^2); b4A$:fdec$; c4A$:afbe$; d4A$:ebfa$; e4A$:acbd$; f4A$:dbca$
4C	13	$2^{1+11}.S_4[.D_8]$	0	$1^{23},-3$	$+^4,-^8,+^{12}$	$-^8,+^{16}\times$ $0^3,4^2,0^{19}$	$2^2.2^3.2^3$	a4A$:cedf$(4A7^2); b4A$:fdec$; c4B$:eafb$(4A6|8); d4B$:bfae$; e4B$:acbd$; f4B$:dbca$

abc	#	$C_M\langle a,b,c\rangle$	wt	a	b	c	$\langle a,b,c\rangle$	net
4D	1	$2^{3+12}.S_3[.S_4]$	0	$+^6, -^8, +^{10}$	$0^{10}, 2^2, 0^6, 2^6$	$+^{12}, -^8, +^4 \times 0^{12}, 2^8, 0^4$	$2^3.2^3$	a4B : $cedf$(2B6^2); b4B : $fdec$; c4B : $eafb$; d4B : $bfae$; e4B : $acbd$; f4B : $dbca$
4D	2	$2^{2+8+4}.D_{10}[.S_4]$	8	$-, +^6, -^7, +^{10}$	$2^2, 0^{11}, 2^6, 0^5$	$-^8, +^{16} \times 2^8, 0^{16}$	$2^2.2^3.2^3$	a4B : $cedf$(4A6^2); b4B : $fdec$; c4B : $eafb$; d4B : $bfae$; e4B : $acbd$; f4B : $dbca$
5A	1	A_{12}	26	12.34	13.24	25.34	A_5	a2A : bc; b3A : cac; c5A : $cbabcS$(3A3)
5A	2	$U_3(8).3$	−28	$AB.CD$	$AB.EF$	$BC.DE$	A_5	a2A : bc; b3C : cac; c5A : $cbabcS$(3C2)
5A	3	S_{10}	−34	12.34	23.67	45.67	S_5	a2A : cd; b3A : dcd; c4B : $adbd$(6A1|21); d6A : $bcacbdT$(4B2|19, 3A1|2^2)
5A	4	$\frac{1}{2}S_6 \wr 2$	−16	12.34	12.35	24.36	A_6	a3A : bcd; b3A : adc; c4B : $abdd$(5A1^2); d5A : $cbacdT$(4B17)
5A	5	$2^6.U_4(2)$	−28	29.38	17.28.39. $4X.5E.6T$	17.29.3X. $4E.5T$.68	$2^4.D_{10}$	a2A : bd; b4B : $adcd$(4A2|11); c4A : $bdcTd$(4B6|16); d5A : $cbabc$(5A6)
5A	6	$2^6.U_4(2)$	−4	2X.48	17.28.39. $4X.5E.6T$	17.29.3X. $4E.5T$.68	$2^4.D_{10}$	a2A : bd; b4A : $adcd$(4B5|16); c4B : $bdcTd$(4B6|15); d5A : $cbabc$(5A5)
5A	7	$2.M_{22}[.2]$	−10	45.XE	18.27.3X. 49.5E.6T	17.29.38. $4X.5E.6T$	$2 \times S_5$	a2A : cf; b2A : dg; c4A : $fafe$(6A3|22); d4A : $gbge$; e6A : $fcfgdg$(10A?^{2}, 2B4|7^2) f6A : $cacege$(4A5|16, 3A1|5^2); g6A : $dbdefe$;
5A	8	$2.M_{21}[.2]$	−28	12.78	18.27.3X. 49.5T.6E	17.29.38. $4X.5E.6T$	$2 \times S_6$	a2A : fg; b4A : $defg$(10A?^{2}); c4A : $edgf$; d4A : $bgce$(6A9|17); e4A : $cfbd$; f6A : $gagbec$(6A4|31, 4A2|16^2); g6A : $fafcdb$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
5A	9	$M_{12}[.2]$	-28	$1X.25.37.$ $48.69.ET$	$12.39.4T.$ $5X.67.8E$	$14.2X.38.$ $59.6E.7T$	$L_2(11)$	$a2\mathrm{A}:fg$; $b3\mathrm{A}:efd$; $c3\mathrm{A}:dge$; $d5\mathrm{A}:gcebf(11\mathrm{A}?\vert?)$; $e5\mathrm{A}:fbdcg$; $f6\mathrm{A}:gagdbe(5\mathrm{A}1\vert10, 6\mathrm{A}1\vert28^2)$; $g6\mathrm{A}:fafecd$
5A	10	$M_{12}[.2]$	-16	$1E.27.35.$ $46.89.XT$	$1T.23.49$ $57.68.EX$	$12.38.46$ $57.9X.ET$	$L_2(11)$	$a2\mathrm{A}:de$; $b3\mathrm{A}:gcf$; $c3\mathrm{A}:fbg$; $d5\mathrm{A}:feaeg(6\mathrm{A}28)$; $e5\mathrm{A}:gdadf$; $f6\mathrm{A}:cgedgb(11\mathrm{A}?^2, 5\mathrm{A}9\vert1^2)$; $g6\mathrm{A}:bfdefc$
5A	11	$\frac{1}{2}S_4 \wr S_3$	-22	b_1b_3	$(b_2b_3)^a$	$(ac_3)^{b_3a}$	$2^4.A_5$	$a2\mathrm{A}:dg$; $b3\mathrm{A}:cfe$; $c4\mathrm{A}:fbeg(4\mathrm{B}12\vert16)$; $d5\mathrm{A}:fgage(6\mathrm{A}26)$; $e5\mathrm{A}:bfdgc(6\mathrm{C}13)$; $f5\mathrm{A}:bcgde$; $g6\mathrm{A}:adfced(5\mathrm{A}1\vert11, 4\mathrm{A}2\vert11^2)$
5A	12	$\frac{1}{2}S_3 \wr A_4[.3]$	-28	z_1z_3	c_1c_2	$z_2^{c_3}z_3$	$3^4.A_5$	$a3\mathrm{A}:egf$; $b3\mathrm{A}:fgd$; $c3\mathrm{A}:dge$; $d5\mathrm{A}:fbgce(9\mathrm{A}?)$; $e5\mathrm{A}:dcgaf$; $f5\mathrm{A}:eagbd$; $g6\mathrm{A}:aecdbf(6\mathrm{D}1\vert7, 5\mathrm{A}1^3)$
5A	13	$2^5.A_6[.2]$	-26	$12.34.5E.$ $6T.78.9X$	$18.27.35.$ $46.9E.XT$	$14.26.3E.$ $59.7X.8T$	$2^5.S_5$	$a3\mathrm{A}:fdg$; $b3\mathrm{A}:gef$; $c4\mathrm{A}:dfeg(4\mathrm{B}10\vert18)$; $d4\mathrm{A}:afcg(6\mathrm{A}5\vert22)$; $e4\mathrm{A}:bgcf$; $f6\mathrm{A}:ecdagb(8\mathrm{B}?\vert?, 6\mathrm{A}18\vert4^2)$; $g6\mathrm{A}:dcebfa$
5A	14	$2^5.S_5[.2,*]$	-24	$17.24.36.$ $5E.8X.9T$	$18.27.35.$ $4X.6T.9E$	$12.34.5E.$ $6T.78.9X$	$2^5.A_6$	$a3\mathrm{A}:cgf$; $b3\mathrm{A}:dge$; $c4\mathrm{A}:afeg(10\mathrm{A}?^2)$; $d4\mathrm{A}:befg$; $e5\mathrm{A}:bgcfd(8\mathrm{B}?)$; $f5\mathrm{A}:agdec$; $g6\mathrm{A}:acebdf(10\mathrm{A}?^2, 4\mathrm{B}2\vert11\vert15)$
5A	15	$L_3(2)\times 2^2[.3]$	-26	$KL.MN.OP.$ $QR.ST.UV.$ $WX.YZ$	$GK.HN.IL.$ $JM.OS.PV.$ $QT.RU$	$FI.GH.KZ.$ $LU.MQ.OV.$ $PY.TX$	$2^2.M_{21}$	$a3\mathrm{A}:efg$; $b4\mathrm{A}:cgfd(10\mathrm{A}?\vert?)$; $c4\mathrm{A}:degb$; $d4\mathrm{A}:bfec$; $e5\mathrm{A}:dfagc(14\mathrm{A}?)$; $f5\mathrm{A}:bgaed$; $g5\mathrm{A}:ceafb$

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abc	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
5A	16	$A_6[.D_{10}]$	-26	12.34	17.28.39. $4X.5E.6T$	17.29.3X. $4E.5T.68$	$A_6 \wr 2$	$a4\text{A}:bfeg(10\text{A}?\|?)$; $b4\text{A}:cfag$; $c4\text{A}:dfbg$; $d4\text{A}:efcg$; $e4\text{A}:afdg$; $f5\text{A}:cdeab(20\text{B}?)$; $g5\text{A}:baedc$
5A	17	$A_7[.D_{10}]$	-29	24.35	17.28.39. $4X.5E.6T$	17.29.3X. $4E.5T.68$	$A_5 \wr 2$	$a4\text{A}:bfeg(6\text{A}7\|23)$; $b4\text{A}:cfag$; $c4\text{A}:dfbg$; $d4\text{A}:efcg$; $e4\text{A}:afdg$; $f5\text{A}:abcde(15\text{A}?)$; $g5\text{A}:edcba$
5A	18	$2^{1+8}.D_{10}[.D_{10}]$	-24	$(\gamma,\delta)(\epsilon,\zeta)$	$(\beta^\omega,\gamma)(\delta^\omega,\epsilon)$	$2(1+i)\times$ $(0^2,1,-i,i,-1)$	$2^{1+8}.D_{10}$	$a4\text{A}:bfeg(4\text{B}10\|16)$; $b4\text{A}:cfag$; $c4\text{A}:dfbg$; $d4\text{A}:efcg$; $e4\text{A}:afdg$; $f5\text{A}:abcde(10\text{B}?)$; $g5\text{A}:edcba$
5B	1	$5^2.4.2^2[.5^2.6]$	-1	$\begin{matrix}0&0&\overline{\omega}\\0&-1&0\\\omega&0&0\end{matrix}$	$\begin{matrix}0&0&-1\\0&-1&0\\-1&0&0\end{matrix}$	$\begin{matrix}-1&0&0\\0&-2&2\omega\theta\\0&2\overline{\omega\theta}&2\end{matrix}$	$U_3(5)$	$\text{hc}(5,5\omega):a^{25}:a^6(8\text{C}?^2,6\text{A}21^3)$
6A	1	$2.Fi_{22}$	190	12.34	12.45	12.67	$2\times S_3$	$a3\text{A}':b$; $b2\text{A}:abT'$
6A	2	$2^2.U_6(2).2$	-2	12.34	56.78	56.89	$2\times S_3$	$a3\text{A}':b$; $b2\text{B}:abT'$
6A	3	$2^2.U_6(2)$	-50	12.34	56.78	12.89	$2^2\times S_3$	$a2\text{A}:bc$; $b2\text{B}:ac$; $c6\text{A}':ab(6\text{A}1\|2,2\text{A}10^3)$
6A	4	$2^{11}.M_{22}$	46	$12.ET$	$23.9T$	$\|b_3e_3\|$	$2\times S_4$	$a2\text{A}:bc$; $b3\text{A}:cac$; $c4\text{A}:abcT'b(4\text{A}3\|4)$
6A	5	$2^{11}.M_{21}.2$	-18	$1T.3X$	$2T.4X$	$\|b_3e_3\|$	$2\times S_4$	$a2\text{B}:bc$; $b3\text{A}:cac$; $c4\text{A}:abcT'b(4\text{A}1\|4)$
6A	6	$L_4(3).2$	-32	12.67	23.67	$\|b_3e_3\|$	$3^2.D_8$	$a2\text{A}':c$; $b3\text{A}':c$; $c4\text{B}:acbc(6\text{A}1\|6)$
6A	7	$2.U_4(3)$	-8	12.45	$78.ET$	$\|b_3e_3\|$	$3^2.D_8$	$a2\text{B}':c$; $b3\text{A}':c$; $c4\text{A}:acbc(6\text{A}2\|7)$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6A	8	$2\times O_8^+(2).3$	22	12.45	23.45	46.57	$\frac{1}{2}(S_3\times D_8)$	$a3\mathrm{A}':c$; $b6\mathrm{A}':c(6\mathrm{A}8^2, 2\mathrm{A}7^3)$; $c4\mathrm{B}:acbc(2\mathrm{A}7\mid 9)$
6A	9	$O_7(3)$	28	12.45	23.45	56.78	$S_3\times S_3$	$a3\mathrm{A}':d$; $b2\mathrm{A}:cd$; $c3\mathrm{A}:dbd$; $d6\mathrm{A}:cbcdad(2\mathrm{A}9\mid 12, 6\mathrm{A}1\mid 9^2)$
6A	10	$G_2(3)$	−26	AB	$BC.DE$	$BC.EF$	$3^{1+2}.2^2$	$a3\mathrm{A}':d$; $b2\mathrm{A}:cd$; $c3\mathrm{C}:dbd$; $d6\mathrm{A}:cbcdad(6\mathrm{A}9\mid 12, 6\mathrm{A}1\mid 10^2)$
6A	11	$2.U_4(3)[.2]$	16	12.34	12.89	$\lvert b_3e_3\rvert$	$2\times 3^2.D_8$	$a2\mathrm{A}:cd$; $b6\mathrm{A}':cd(6\mathrm{A}7^2, 2\mathrm{A}13^3)$; $c4\mathrm{A}:adbd(6\mathrm{A}3\mid 7)$; $d4\mathrm{A}:acbc$
6A	12	$3\times G_2(3)[.2]$	−14	$AB.DE$	$AB.EF$	$BC.EF$	$3^{1+2}.2$	$a3\mathrm{A}:dbc$; $b3\mathrm{A}:cad$; $c3\mathrm{C}:bda$; $d3\mathrm{C}:acb$
6A	13	$2^2.U_4(3)[.S_3]$	4	12.89	17.28.35. 46.9X.ET	18.27.36. 45.9X.ET	$2\times S_3\times S_3$	$a2\mathrm{B}:de$; $b2\mathrm{B}:de$; $c2\mathrm{B}:de$; $d6\mathrm{A}:aebece(6\mathrm{A}2\mid 9, 6\mathrm{A}3^3)$; $e6\mathrm{A}:cdbdad$
6A	14	$2^{6+4}.S_6[.2]$	−18	1T.34	12.39	$\lvert b_3e_3\rvert$	$2^4.D_{12}$	$a3\mathrm{A}:ecd$; $b3\mathrm{A}:dce$; $c4\mathrm{A}:aebd(4\mathrm{A}4\mid 7)$; $d4\mathrm{B}:acbe(4\mathrm{B}2\mid 6)$; $e4\mathrm{B}:adbc$
6A	15	$2^{3+6}.L_3(2).2$	−10	ac_1	ad_1	πb_2b_3	$\frac{1}{2}(D_8\times 2^4.S_3)$	$a3\mathrm{A}:dce$; $b6\mathrm{A}':ecd(6\mathrm{A}15^2, 2\mathrm{B}6\mid 4^2)$; $c4\mathrm{B}:adbe(4\mathrm{A}5\mid 8)$; $d4\mathrm{A}:aebc(4\mathrm{B}7\mid 12)$; $e4\mathrm{A}:acbd$
6A	16	$2^7.S_6(2)[.2]$	−26	12.34	12.56	23.78	$2\times S_4$	$a2\mathrm{A}:ce$; $b2\mathrm{A}:de$; $c4\mathrm{B}:aede(4\mathrm{B}1\mid 12)$; $d4\mathrm{B}:bece$; $e6\mathrm{A}:cacdbd(3\mathrm{A}2^2, 2\mathrm{B}1\mid 6^2)$
6A	17	$2^{1+10}.S_6.2[.2]$	6	1T.67	2E.56	$\lvert b_3e_3\rvert$	$2^2\times S_4$	$a2\mathrm{B}:ce$; $b2\mathrm{B}:de$; $c4\mathrm{B}:aede(4\mathrm{B}5\mid 12)$; $d4\mathrm{B}:bece$; $e6\mathrm{A}:cacdbd(6\mathrm{C}2^2, 2\mathrm{B}2\mid 6^2)$
6A	18	$2^{11}.M_{21}$	−2	12.56	12.34.56. 78.9X.ET	15.27.36. 48.9E.XT	$2^2\times S_4$	$a2\mathrm{A}:ce$; $b2\mathrm{B}:de$; $c4\mathrm{A}:aede(4\mathrm{A}3\mid 5)$; $d4\mathrm{A}:bece(4\mathrm{A}1\mid 5)$; $e6\mathrm{A}:acdbdc(6\mathrm{A}4\mid 5, 2\mathrm{A}10\mid 13^2)$
6A	19	$2^{6+4}.A_7$	−2	12.34	12.XT	$\lvert b_3e_3\rvert$	$2^4.D_{12}$	$a2\mathrm{A}:ce$; $b4\mathrm{A}':de(4\mathrm{A}5\mid 8)$; $c4\mathrm{A}:aede(4\mathrm{B}5\mid 18)$; $d4\mathrm{B}:bece(4\mathrm{A}8\mid 13)$; $e6\mathrm{A}:cacdbd(3\mathrm{A}2\mid 6, 4\mathrm{A}2\mid 9^2)$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6A	20	$2^{6+4}.A_6$	-26	45.67	$\vert d_3a_1\vert$	$\vert b_3e_3\vert$	$2^4.(2^2\times S_3)$	$a2\mathrm{A}{:}ce$; $b4\mathrm{A}'{:}de(4\mathrm{A}5\vert 8)$; $c4\mathrm{B}{:}aede(4\mathrm{A}2\vert 13)$; $d4\mathrm{A}{:}bece(4\mathrm{B}6\vert 18)$; $e6\mathrm{A}{:}cacdbd(6\mathrm{A}4\vert 14, 4\mathrm{A}2\vert 9^2)$
6A	21	S_{10}	-20	12.34	13.24	45.67	S_5	$a2\mathrm{A}'{:}e$; $b2\mathrm{A}{:}cd$; $c4\mathrm{B}{:}bded(3\mathrm{A}1\vert 3)$; $d5\mathrm{A}{:}ecbce(4\mathrm{B}19)$; $e6\mathrm{A}{:}dcdeae(5\mathrm{A}1\vert 3, 4\mathrm{B}1\vert 19^2)$
6A	22	$2.M_{22}$	4	$45.XE$	$18.27.3X.$ $49.5E.6T$	$19.2X.37.$ $48.5E.6T$	$2\times S_5$	$a2\mathrm{B}'{:}e$; $b2\mathrm{A}{:}cd$; $c4\mathrm{A}{:}bded(6\mathrm{A}3\vert 23)$; $d5\mathrm{A}{:}ecbce(4\mathrm{A}15)$; $e6\mathrm{A}{:}dcdeae(10\mathrm{A}?\vert ?, 4\mathrm{A}3\vert 15^2)$
6A	23	$2.M_{22}.2[.2]$	0	$18.27.3X.$ $49.5E.6T$	$19.2X.37.$ $48.5E.6T$	$17.29.38.$ $4E.5X.6T$	$2\times A_5$	$a2\mathrm{B}{:}de$; $b3\mathrm{A}{:}dce$; $c3\mathrm{A}{:}ebd$; $d5\mathrm{A}{:}beaec(10\mathrm{A}?)$; $e5\mathrm{A}{:}cdadb$
6A	24	$3^{1+6}.2^{1+4}.3[.A_4]$	-26	$z_1^{c_2}f_1$	$z_2^{c_3}f$	$z_3^{c_1}a$	$3^{1+2}.2\times S_3$	$a3\mathrm{A}{:}fgh$; $b3\mathrm{A}{:}ehg$; $c3\mathrm{A}{:}hef$; $d3\mathrm{A}{:}gfe$; $e6\mathrm{A}{:}hbgdfc(6\mathrm{D}1\vert 5, 6\mathrm{A}9^3)$; $f6\mathrm{A}{:}gahced$; $g6\mathrm{A}{:}fdebha$; $h6\mathrm{A}{:}ecfagb$
6A	25	$3^4.2^{1+4}.3^2[.S_3]$	-20	z_1z_3	c_1c_2	$z_2^{c_3}z_3^{c_2}$	$3^4.D_{10}$	$a3\mathrm{A}{:}cde$; $b3\mathrm{A}{:}hgf$; $c5\mathrm{A}{:}hdaeg(6\mathrm{D}7)$; $d5\mathrm{A}{:}feach$; $e5\mathrm{A}{:}gcadf$; $f5\mathrm{A}{:}dhbge$; $g5\mathrm{A}{:}efbhc$; $h5\mathrm{A}{:}cgbfd$
6A	26	$\frac{1}{2}S_4\wr S_3[.2^2]$	-8	$b_1^{c_1}b_3$	$(b_1b_2)^a$	$(b_1b_2)^{ab_1}$	$2^4.A_5$	$a2\mathrm{A}{:}cd$; $b2\mathrm{A}{:}ef$; $c5\mathrm{A}{:}gdadh(5\mathrm{A}11)$; $d5\mathrm{A}{:}hcacg$; $e5\mathrm{A}{:}hfbfg$; $f5\mathrm{A}{:}gebeh$; $g6\mathrm{A}{:}chefhd(6\mathrm{C}13^2, 4\mathrm{B}12\vert 15^2)$; $h6\mathrm{A}{:}dgfegc$
6A	27	$2^6.A_7[.2^2]$	-20	23.67	$14.25.36.$ $78.9X.ET$	$15.24.36.$ $78.9X.ET$	$\frac{1}{2}S_4\wr 2$	$a2\mathrm{A}{:}ef$; $b2\mathrm{A}{:}gh$; $c4\mathrm{A}{:}efgh(4\mathrm{B}18^2)$; $d4\mathrm{A}{:}fehg$; $e6\mathrm{A}{:}fafchd(6\mathrm{A}4\vert 32, 3\mathrm{A}1\vert 5^2)$; $f6\mathrm{A}{:}eaedgc$; $g6\mathrm{A}{:}hbhcfd$; $h6\mathrm{A}{:}gbgdec$
6A	28	$M_{12}[.2]$	-14	$1E.27.35.$ $46.89.XT$	$1T.23.49.$ $57.68.XE$	$18.29.3E.$ $46.5T.7X$	$L_2(11)$	$a2\mathrm{A}{:}cg$; $b2\mathrm{A}{:}dh$; $c5\mathrm{A}{:}egagf(5\mathrm{A}10)$; $d5\mathrm{A}{:}fhbhe$; $e5\mathrm{A}{:}gcfdh(11\mathrm{A}?)$; $f5\mathrm{A}{:}hdecg$; $g6\mathrm{A}{:}cacehf(5\mathrm{A}1\vert 9, 3\mathrm{A}1\vert 3^2)$; $h6\mathrm{A}{:}dbdfge$

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abc	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6A	29	$2^6.S_5[.2]$	−16	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$13.28.4E.$ $5X.6T.79$	$2^5.A_5$	a3A : dhe; b3A : fhg; c4A : $defg$(4B16^2); d5A : $aecgh$(10A?); e5A : $ahfcd$; f5A : $bgceh$; g5A : $bhdcf$; h6A : $eadgbf$(6A23^2, 4B2$\vert$15^2)
6A	30	$2^5.A_6[.2]$	−20	$17.28.3X.$ $49.56.ET$	$12.39.4X.$ $5T.6E.78$	$14.25.36.$ $7X.8E.9T$	$2^5.S_5$	a2A : gh; b4A : $cfde$(4B10$\vert$16); c4A : $begf$(6A18$\vert$23); d4A : $bfhe$; e5A : $gcbdh$(8B?); f5A : $hdbcg$; g6A : $hahfce$(5A1$\vert$13, 4A2$\vert$15^2); h6A : $gagedf$
6A	31	$2.M_{21}$	−14	$56.ET$	$18.27.3X.$ $49.5E.6T$	$17.29.38.$ $4E.5X.6T$	$2\times S_6$	a2A : ef; b3A : gch; c4A : $bgdh$(6A5$\vert$13); d4A : $cgfh$(6A18$\vert$22); e5A : $hfafg$(6A31); f6A : $eaehdg$(4A5$\vert$12, 4A2$\vert$15^2); g6A : $hefdcb$(10A?$\vert$?, 6A9$\vert$18$\vert$22); h6A : $cdfegb$
6A	32	$2^6.A_7$	−20	$12.XT$	$34.ET$	$\vert b_3e_3\vert$	$\frac{1}{2}S_4\wr 2$	a2A : ef; b3A : gdh; c3A : heg; d4A : $bgfh$(4B12$\vert$18); e6A : $fafgch$(4A4$\vert$14, 6A1$\vert$32^2); f6A : $eaehdg$(6A9$\vert$32, 4A2$\vert$13^2); g6A : $cefdbh$(4B13$\vert$18, 6A4$\vert$9$\vert$32); h6A : $gbdfec$
6A	33	$3^4.A_6[.D_{12}]$	−11	12.45	23.78	$\vert b_3e_3\vert$	$3^4.2^{1+3}$	a4A : $bgfh$(6A7$\vert$13); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(6A7^2, 6A11^3); h6A : $fedcba$
6A	34	$2^{6+8}.A_6[.D_{12}]$	6	$1E.2T.39.$ $4X.56.78$	$13.24.5T.$ $6E.7X.89$	$12.34.57.$ $68.9T.XE$	$2^4.D_{12}$	a4A : $bgfh$(4A5$\vert$8); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(3A6^2, 2B4^3); h6A : $fedcba$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6A	35	$A_7[.D_{12}]$	-19	37.59	12.34.56. $78.9X.ET$	12.36.45. $7X.9T.8E$	$\frac{1}{2}S_5 \wr 2$	a4A : $bgfh$(6A7|22); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(15A?^{2}, 4B11^3); h6A : $fedcba$
6A	36	$2^{6+8}.S_4[.D_{12}]$	-26	$\omega \times 5, 1^{23}$	$-2^4, 2^4, 0^{16}$	$-1, 3, -1^6, 1^{16}$	$2^2 \times 2^4.S_3$	a4A : $bgfh$(4A5|8); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(6C8^2, 2B4^3); h6A : $fedcba$
6A	37	$2^{1+9}.3^2.2[.D_{12}]$	-14	34.57	b_3z_2	12.39.4X. 58.67.ET	$(2^{1+4}.S_3 \times 2^2).2$	a4A : $bgfh$(4B10|14); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(12A?^{2}, 4A8^3); h6A : $fedcba$
6A	38	$2^9.3^2.2[.D_{12}]$	-22	15.34	$\lvert b_3e_3\rvert$	12.3X|49. ET|58.67	$2^{1+8}.D_{12}$	a4A : $bgfh$(4B10|18); b4A : $cgah$; c4A : $dgbh$; d4A : $egch$; e4A : $fgdh$; f4A : $ageh$; g6A : $abcdef$(6C8^2, 4B11^3); h6A : $fedcba$
6B	1	$U_4(2)$	-13	12.49	23.45	$\lvert b_3e_3\rvert$	$(S_3^2 \times A_4).2$	a3A : cde; b6A$'$: edc(6C10^2, 6A6|11^2); c4B : $aebd$(6A9|11); d4A : $acbe$(12C?|?); e4A : $adbc$
6B	2	$2^{4+4}.A_4[.A_4]$	-23	18.29.3X. 45.67.ET	$\lvert d_2g_2\rvert$	$\lvert d_2g_2\rvert^{(123)}$	$2 \times 2^6.3^{1+2}.2$	a3A : fgh; b3A : ehg; c3A : hef; d3A : gfe; e6A : $hbgdfc$(6D1|8, 4A5^3); f6A : $gahced$; g6A : $fdebha$; h6A : $ecfagb$

abc	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6B	3	$Q_8 \circ 2A_4[.S_3]$	-17	$d_1 b_2^{ab_3c_3}$	$c_1 b_2^{ab_3c_3}$	$(18.36.57.9X.2T.4E)^{fm2m}$	$2^{1+10}.3^4.D_{10}$	a3A : cde; b3A : hgf; c5A : $hdaeg$(12C?); d5A : $feach$; e5A : $gcadf$; f5A : $dhbge$; g5A : $efbhc$; h5A : $cgbfd$
6B	4	$2^{4+4}.A_5[.2^2]$	-7	$12.34.59.6X.7E.8T$	$12.34.59.6E.7T.8X$	$15.26.37.48.9X.ET$	$2^6.3^{1+2}.2$	a3A : ebf; b3A : fae; c3A : gdh; d3A : hcg; e6A : $hfbafg$($6\mathrm{B}4^2, 4\mathrm{A}10\vert 5^2$); f6A : $geabeh$; g6A : $fhdche$; h6A : $egcdgf$
6B	5	$3^3.A_4[.2]$	-19	$(c_1g_1)^{b_1}$	$(ac_1)^{b_1c_1z_2}$	$az_3^{c_1}$	$3^5.2^4.A_5$	a3A : dhe; b3A : fhg; c4A : $defg$($12\mathrm{A}?^2$); d5A : $aecgh$(15A?); e5A : $hfcda$; f5A : $bgceh$; g5A : $hdcfb$; h6A : $eadgbf$($9\mathrm{A}?^2, 4\mathrm{B}2\vert 15^2$)
6B	6	S_5	-19	$12.34.56.78.9X.ET$	$12.34.56.7X.9T.8E$	$12.37.49.5E.68.XT$	S_{10}	a3A : fhg; b4A : $chde$($10\mathrm{A}?\vert ?$); c4A : $begh$($8\mathrm{B}?\vert ?$); d4A : $hfeb$; e5A : $gcbdf$(20B?); f5A : $agedh$(14A); g5A : $hcefa$; h6A : $afdbcg$($14\mathrm{A}?\vert ?, 6\mathrm{A}4\vert 22^2$)
6B	7	A_6	-17	13.48	$12.34.56.78.9X.ET$	$12.36.4E.7X.9T.58$	$\frac{1}{2}S_6 \wr 2$	a3A : gbh; b4A : $agfh$($10\mathrm{A}?\vert ?$); c4A : $eghd$($12\mathrm{C}?^2$); d4A : $chfe$($8\mathrm{B}?\vert ?$); e4A : $dfgc$; f5A : $dhbge$(12C?); g6A : $ahcefb$($10\mathrm{A}?\vert ?, 6\mathrm{A}4\vert 11\vert 31$); h6A : $bfdcga$
6B	8	$L_2(11)[.2]$	-21	$17.2E.3T.48.56.9X$	$1E.23.45.6X.7T.89$	$14.29.35.6X.7E.8T$	M_{12}	a3A : geh; b3A : hfg; c4A : $degf$($10\mathrm{A}?\vert ?$); d4A : $cfhe$; e5A : $dhagc$(11A?); f5A : $cgbhd$; g6A : $ahbfce$($8\mathrm{B}?\vert ?, 5\mathrm{A}7\vert 1^2$); h6A : $bgaedf$

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abc	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6B	9	$L_2(11)[.2,*]$	-15	$1E.26.3T.$ $45.78.9X$	$13.2T.47.$ $59.6X.8E$	$15.26.3E.$ $47.8X.9T$	M_{12}	$a3\mathrm{A}:ceg$; $b3\mathrm{A}:dfh$; $c4\mathrm{A}:aghe(\mathrm{8B?}\vert?)$; $d4\mathrm{A}:bhgf$; $e5\mathrm{A}:achfg(\mathrm{8B?})$; $f5\mathrm{A}:bdgeh$; $g6\mathrm{A}:aefdhc(\mathrm{11A?}\vert?,\mathrm{6A4}\vert 22\vert 28)$; $h6\mathrm{A}:bfecgd$
6B	10	$A_6[.D_{12}]$	-20	17.35	$\vert b_3e_3\vert$	$\vert d_3a_1\vert$	$\frac{1}{2}S_6\wr 2$	$a4\mathrm{A}:bgfh(\mathrm{10A?}^2)$, $b4\mathrm{A}:cgah$; $c4\mathrm{A}:dgbh$; $d4\mathrm{A}:egch$; $e4\mathrm{A}:fgdh$; $f4\mathrm{A}:ageh$; $g6\mathrm{A}:abcdef(\mathrm{12C?}^2,\mathrm{4B11}^3)$; $h6\mathrm{A}:fedcba$
6B	11	$A_6[.6,\$]$	-18	19.35	$\vert b_3e_3\vert$	$\vert d_3a_1\vert$	$\frac{1}{2}S_6\wr 2$	$a4\mathrm{A}:bgfh(\mathrm{8B?}^2)$, $b4\mathrm{A}:cgah$; $c4\mathrm{A}:dgbh$; $d4\mathrm{A}:egch$; $e4\mathrm{A}:fgdh$; $f4\mathrm{A}:ageh$; $g6\mathrm{A}:abcdef(\mathrm{15A?}^2,\mathrm{6A11}^3)$; $h6\mathrm{A}:fedcba$
6B	12	$2^4.D_{10}[.2^2]$	-17	$100\bar{1}0.01\bar{1}00$	$0100\bar{1}.001\bar{1}0$	$00011.\bar{1}111\bar{1}$	$2^6.U_4(2)$	$a4\mathrm{A}:bfhe(\mathrm{12C?}\vert?)$; $b4\mathrm{A}:aegf$; $c4\mathrm{A}:dgeh$; $d4\mathrm{A}:chfg$; $e5\mathrm{A}:ahcgb(\mathrm{12F?})$; $f5\mathrm{A}:bgdha$; $g5\mathrm{A}:dfbec$; $h5\mathrm{A}:ceafd$
6B	13	$2.S_4[.2^2]$	-20	$34.z_3$	$12.37.45.$ $6E.8T.9X$	$12.35.46.$ $79.8X.ET$	$2\times {}^2F_4{}'(2)$	$a4\mathrm{A}:bfhe(\mathrm{10B?}\vert?)$; $b4\mathrm{A}:aegf$; $c4\mathrm{A}:dgeh$; $d4\mathrm{A}:chfg$; $e5\mathrm{A}:ahcgb(\mathrm{13A?})$; $f5\mathrm{A}:bgdha$; $g5\mathrm{A}:dfbec$; $h5\mathrm{A}:ceafd$
6B	14	$2.S_4[.2^2,*]$	-18	$12.37.45.$ $6E.8T.9X$	$1E.39.46.$ $5X.78.9T$	$12.35.46.7X.$ $89.ET.z_1z_2$	$2\times {}^2F_4{}'(2)$	$a4\mathrm{A}:bfhe(\mathrm{8B?}\vert?)$; $b4\mathrm{A}:aegf$; $c4\mathrm{A}:dgeh$; $d4\mathrm{A}:chfg$; $e5\mathrm{A}:ahcgb(\mathrm{16A?})$; $f5\mathrm{A}:bgdha$; $g5\mathrm{A}:dfbec$; $h5\mathrm{A}:ceafd$
6C	1	$2^{3+8}.U_4(2).2[.2]$	14	$12.XE$	58.67	$\vert b_3e_3\vert$	$2^2\times S_3$	$a2\mathrm{B}:bc$; $b2\mathrm{B}:ac$; $c6\mathrm{A}':ab(\mathrm{6A2}^2,\mathrm{2B2}^3)$
6C	2	$2^{1+10}.(S_6\times S_3)$	-10	15.24	16.34	25.78	$2\times S_4$	$a2\mathrm{B}:bc$; $b3\mathrm{A}:cac$; $c4\mathrm{B}:abcT'b(\mathrm{4B2}\vert 5)$
6C	3	$2\times U_4(2).2[.2]$	8	12.58	$67.XE$	$\vert b_3e_3\vert$	$2\times 3^2.D_8$	$a2\mathrm{B}:cd$; $b6\mathrm{A}':cd(\mathrm{6A6}^2,\mathrm{2B6}^3)$; $c4\mathrm{B}:adbd(\mathrm{6A3}\vert 6)$; $d4\mathrm{B}:acbc$
6C	4	$2^{1+14}.S_5[.2]$	-2	13.24	$15.26.37.$ $48.9X.ET$	$12.38.47.$ $56.9X.ET$	$2^2\times S_4$	$a2\mathrm{B}:ce$; $b2\mathrm{B}:de$; $c4\mathrm{A}:aede(\mathrm{4A1}\vert 5)$; $d4\mathrm{A}:bece$; $e6\mathrm{A}:cacdbd(\mathrm{6A5}^2,\mathrm{2B2}\vert 4^2)$

$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6C	5	$2^{10}.2^3.3^2.2[.2]$	6	16.25	$45.b_3$	$\lvert d_3g_3\rvert$	$2^2\times S_4$	$a\mathrm{2B}:ce$; $b\mathrm{2B}:de$; $c\mathrm{4B}:aede(\mathrm{4B5}\mid 12)$; $d\mathrm{4B}:bece$; $e\mathrm{6A}:cacdbd(\mathrm{6C2}^2,\mathrm{2B2}\mid 6^2)$
6C	6	$2^{10}.S_5$	6	13.57	15.26.38. $47.9X.ET$	13.24.58. $67.9X.ET$	$2^5.D_{12}$	$a\mathrm{2B}:ce$; $b\mathrm{4A}':de(\mathrm{4A5}\mid 8)$; $c\mathrm{4B}:aede(\mathrm{4A3}\mid 13)$; $d\mathrm{4A}:bece(\mathrm{4B6}\mid 18)$; $e\mathrm{6A}:cacdbd(\mathrm{6A5}\mid 14,\mathrm{4A3}\mid 9^2)$
6C	7	$2^{10}.\frac{1}{2}(S_4\times S_3)$	-2	z_1a	$z_1^*b_2d_2$	$z_1^*b_2d_2ff_1f_2f_3$	$2^5.D_{12}$	$a\mathrm{2B}:ce$; $b\mathrm{4A}':de(\mathrm{4A5}\mid 7)$; $c\mathrm{4A}:aede(\mathrm{4B4}\mid 18)$; $d\mathrm{4B}:bece(\mathrm{4A8}\mid 13)$; $e\mathrm{6A}:cacdbd(\mathrm{6C2}\mid 8,\mathrm{4A3}\mid 9^2)$
6C	8	$2^{1+13}.2^2.3^2.2[.S_3]$	22	$b_1d_1b_2d_3f_3z_1$	$b_1^*b_2b_3d_1f$	$b_1^*b_2b_3c_1f$	$2^5.S_3$	$a\mathrm{3A}:cde$; $b\mathrm{3A}:edc$; $c\mathrm{4A}:aebd(\mathrm{4A4}\mid 8)$; $d\mathrm{4A}:acbe$; $e\mathrm{4A}:adbc$
6C	9	$2^{4+6}.S_4[.3]$	-2	$b_2z_1b_1d_1d_3f_3$	$(b_1^*b_2)^{c_2}b_3fd_1$	$(b_1^*b_2)^{c_2}b_3fc_1$	$\frac{1}{2}(D_8\times 2^4.S_3)$	$a\mathrm{3A}:cde$; $b\mathrm{6A}':edc(\mathrm{6C9}^2,\mathrm{2B6}^3)$; $c\mathrm{4B}:aebd(\mathrm{4A5}\mid 6)$; $d\mathrm{4B}:acbe$; $e\mathrm{4B}:adbc$
6C	10	$2\times 2^6.U_4(2)$	14	12.67	12.78	$\lvert b_3e_3\rvert$	$\frac{1}{2}(D_8\times S_4)$	$a\mathrm{3A}:cde$; $b\mathrm{6A}':edc(\mathrm{6C10}^2,\mathrm{2A7}\mid 6^2)$; $c\mathrm{4B}:aebd(\mathrm{2A6}\mid 9)$; $d\mathrm{4A}:acbe(\mathrm{4B6}\mid 12)$; $e\mathrm{4A}:adbc$
6C	11	$2^6.A_6[.2^2]$	4	12.34	13.25.47. $68.9E.XT$	13.25.48. $67.9T.XE$	$2\times\frac{1}{2}S_4\wr 2$	$a\mathrm{2B}:ef$; $b\mathrm{2B}:gh$; $c\mathrm{4A}:efgh(\mathrm{4B18}^2)$; $d\mathrm{4A}:fehg$; $e\mathrm{6A}:fafchd(\mathrm{6A5}\mid 32,\mathrm{6A3}\mid 18^2)$; $f\mathrm{6A}:eaedgc$; $g\mathrm{6A}:hbhcfd$; $h\mathrm{6A}:gbgdec$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6C	12	$2\times 2^5.S_5[.2^2]$	0	$16.25.34.$ $78.9T.XE$	$12.3T.4E.$ $5X.69.78$	$12.39.4X.$ $5E.6T.78$	$2\times 2^4.A_5$	$a2\mathrm{B}:cd$; $b2\mathrm{B}:ef$; $c5\mathrm{A}:gdadh(10\mathrm{A}?)$; $d5\mathrm{A}:hcacg$; $e5\mathrm{A}:gfbfh$; $f5\mathrm{A}:hebeg$; $g6\mathrm{A}:chfehd(6\mathrm{A}29^2, 4\mathrm{B}12\|15^2)$; $h6\mathrm{A}:dgefgc$
6C	13	$\frac{1}{2}S_4\wr S_3[.2]$	8	c_1e_1	c_1f_1	$d_1f_1f^*$	$2^4.A_5$	$a3\mathrm{A}:dhe$; $b3\mathrm{A}:fhg$; $c4\mathrm{A}:defg(4\mathrm{B}16^2)$; $d5\mathrm{A}:aecgh(5\mathrm{A}11)$; $e5\mathrm{A}:hfcda$; $f5\mathrm{A}:bgceh$; $g5\mathrm{A}:hdcfb$; $h6\mathrm{A}:eadgbf(3\mathrm{A}3^2, 4\mathrm{B}2\|15^2)$
6C	14	$2.2^4.S_5[.2]$	4	$KO.LP.MQ.$ $NR.SW.TX.$ $UY.VZ$	$KS.LT.MU.$ $NV.OW.PX.$ $QY.RZ$	$EF.GK.HL.$ $IN.JM.QR.$ $TU.XZ$	$2.2^4.S_5$	$a2\mathrm{B}:gh$; $b4\mathrm{A}:cfde(4\mathrm{B}9\|16)$; $c4\mathrm{B}:begf(3\mathrm{A}3\|5)$; $d4\mathrm{B}:bfhe$; $d4\mathrm{B}:bfhe$; $e5\mathrm{A}:gcbdh(8\mathrm{B}?)$; $f5\mathrm{A}:hdbcg$; $g6\mathrm{A}:hahfce(10\mathrm{A}?\|?, 4\mathrm{B}5\|19^2)$; $h6\mathrm{A}:gagedf$
6C	15	$S_6\times S_4$	2	15.24	16.25	34.78	S_6	$a2\mathrm{B}:ef$; $b3\mathrm{A}:gch$; $c4\mathrm{B}:bgdh(3\mathrm{A}2\|4)$; $d4\mathrm{B}:cgfh(6\mathrm{A}16\|21)$; $e5\mathrm{A}:hfafg(6\mathrm{C}15)$; $f6\mathrm{A}:eaehdg(4\mathrm{B}12\|17, 4\mathrm{B}5\|19^2)$; $g6\mathrm{A}:bhefdc(5\mathrm{A}3\|4, 6\mathrm{A}9\|16\|21)$; $h6\mathrm{A}:cdfegb$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6C	16	$2^5{\cdot}\frac{1}{2}(S_4\times S_3)[.S_3]$	4	$b_1b_2d_1d_3f_3z_1$	$a^{fb_2b_3ac_2c_3}d_1$	$a^{fb_2b_3ac_2c_3}c_1$	$2^{1+8}.S_3^2$	$a\mathrm{3A}:ijk$; $b\mathrm{3A}:nml$; $c\mathrm{4A}:glhi(\mathrm{4B}14^2)$; $d\mathrm{4A}:hmfj$; $e\mathrm{4A}:fngk$; $f\mathrm{6A}:jdmnek(\mathrm{6A}32^2,\mathrm{4A}8\vert 13^2)$; $g\mathrm{6A}:kenlci$; $h\mathrm{6A}:iclmdj$; $i\mathrm{6A}:jakgch(\mathrm{4A}13\vert 14,\mathrm{6A}4\vert 32^2)$; $j\mathrm{6A}:kaihdf$; $k\mathrm{6A}:iajfeg$; $l\mathrm{6A}:nbmhcg$; $m\mathrm{6A}:lbnfdh$; $n\mathrm{6A}:mblgef$
6C	17	$2^{6+8}.3^2.2[.D_{12}]$	6	$-1^7,3,1^{16}$	$2^4,-2^4,0^{16}$	$\omega\times 5,1^{23}$	$2^2\times 2^4.S_3$	$a\mathrm{4A}:bgfh(\mathrm{4A}5\vert 8)$; $b\mathrm{4A}:cgah$; $c\mathrm{4A}:dgbh$; $d\mathrm{4A}:egch$; $e\mathrm{4A}:fgdh$; $f\mathrm{4A}:ageh$; $g\mathrm{6A}:abcdef(\mathrm{6C}8^2,\mathrm{2B}13^3)$; $h\mathrm{6A}:fedcba$
6C	18	$2^7.\frac{1}{2}(S_4\times S_3)[.D_{12}]$	10	$13.b_3$	$1X.24.3T.$ $56.78.9E$	$12.3E.4T.$ $56.78.9X$	$2^{1+8}.D_{12}$	$a\mathrm{4A}:bgfh(\mathrm{4B}10\vert 18)$; $b\mathrm{4A}:cgah$; $c\mathrm{4A}:dgbh$; $d\mathrm{4A}:egch$; $e\mathrm{4A}:fgdh$; $f\mathrm{4A}:ageh$; $g\mathrm{6A}:abcdef(\mathrm{3A}6^2,\mathrm{4B}11^3)$; $h\mathrm{6A}:fedcba$
6C	19	$2^{10}.(2\times S_4)[.2^2]$	-2	$2^6,-2^2,0^{16}$	$-1^8,-3,1^{15}$	$\omega\times 1^{23},5$	$2^2\times 2^4.S_3$	$a\mathrm{4A}:cgdh(\mathrm{4A}5\vert 7)$; $b\mathrm{4A}:egfh$; $c\mathrm{4B}:fgah(\mathrm{4B}6\vert 12)$; $d\mathrm{4B}:ageh$; $e\mathrm{4B}:dgbh$; $f\mathrm{4B}:bgch$; $g\mathrm{6A}:dacfbe(\mathrm{6A}14^2,\mathrm{2B}4\vert 6^2)$; $h\mathrm{6A}:cadebf$
6C	20	$2^{1+9}.D_{10}[.2^2]$	2	$d_1d_2d_3fb_2^*z_2^*$	$\vert b_3e_3\vert$	$13.24.5E.$ $6T.79.8X$	$(2^{1+4}.3\ o$ $2^{1+3}).2$	$a\mathrm{4A}:cgdh(\mathrm{4B}9\vert 14)$; $b\mathrm{4A}:egfh$; $c\mathrm{4B}:fgah(\mathrm{4A}9\vert 10)$; $d\mathrm{4B}:ageh$; $e\mathrm{4B}:dgbh$; $f\mathrm{4B}:bgch$; $g\mathrm{6A}:dacfbe(\mathrm{12A}?^2,\mathrm{4A}8\vert 6^2)$; $h\mathrm{6A}:cadebf$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6C	21	$2^7.D_{12}[.2^2]$	2	$b_3^a d_1$	$b_2^{ab_3c_3z_2}.$ $b_1^{ab_2c_1z_2}$	$b_1^{c_1d_1}.$ $a^{b_3c_3z_2c_1b_1}$	$2^{1+8}.D_{12}$	a4A$:cgdh$(4B9$\vert$18); b4A$:egfh$; c4B$:fgah$(4A9$\vert$13); d4B$:ageh$; e4B$:dgbh$; f4B$:bgch$; g6A$:dacfbe$(6A14^2, 4B11$\vert$8^2); h6A$:cadebf$
6D	1	$3^{1+8}.2^{4+4}.A_4[.A_4]$	49	$c_2^{z_3}f$	$c_3^{z_1}f$	$c_1^{z_2}f$	$3^{1+2}.2$	a3A$:bcd$; b3A$:adc$; c3A$:dab$; d3A$:cba$
6D	2	$\{3^9\}.2A_4[.3]$	-5	$\vert b_1e_1\vert$	$\vert b_2e_2\vert$	$\vert b_1e_1\vert^{e_3f_3}$	$3^{1+2}.2$	a3A$:bcd$; b3C$:adc$; c3C$:dab$; d3C$:cba$
6D	3	$3^5.2\mathrm{A}_6[.2^2,\$]$	4	$1000000\vert 0.$ $0100000\vert 0$	$1011100\vert 0.$ $010000\bar{1}\vert\bar{1}$	$1000001\vert 1.$ $0111100\vert 0$	$3^{1+2}.2$	a3C$:bcd$; b3C$:adc$; c3C$:dab$; d3C$:cba$
6D	4	$3^4.2^{1+4}.3^2.2[.3]$	-5	$ET.b_3$	$\vert d_3g_3\vert$	$14.26.35.$ $7T.8X.9E$	$3^3.D_8$	a3A$:cde$; b6A$'$$:edc$(6A8^2, 6A6^3); c4B$:aebd$(6A6$\vert$9); d4B$:acbe$; e4B$:adbc$
6D	5	$3^{1+6}.2^{4+4}.3[.A_4]$	1	$c_2^{z_3}f$	$c_3^{z_1}f$	$c_1^{z_2}f_1$	$2\times 3^{1+2}.2$	a3A$:fgh$; b3A$:ehg$; c3A$:hef$; d3A$:gfe$; e6A$:bgdfch$(6D1$\vert$5, 2A9^3); f6A$:ahcedg$; g6A$:debhaf$; h6A$:cfagbe$
6D	6	$\{3^5\}.2A_4[.3]$	1	$\vert b_1e_1\vert^{f_2z_2d_2e_2}$	$\vert b_2e_2\vert$	$\vert z_1a\vert$	$3^{2+1+2}.2^2$	a3A$:fgh$; b3C$:ehg$; c3C$:efh$; d3C$:egf$; e6A$:bgdfch$(6D2$\vert$5, 6A10^3); f6A$:ahcedg$(6A12$\vert$24, 6A9$\vert$10^2); g6A$:debhaf$; h6A$:cfagbe$
6D	7	$3^4.2^{1+4}.3^2[.S_3]$	7	z_1z_3	c_1c_2	$c_2c_3^{z_2z_1c_2z_3}$	$3^4.D_{10}$	a3A$:cde$; b3A$:hgf$; c5A$:hdaeg$(6A25); d5A$:feach$; e5A$:gcadf$; f5A$:dhbge$; g5A$:efbhc$; h5A$:cgbfd$

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abc	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6D	8	$2^{1+6}.2A_4[.A_4]$	1	$\lvert d_2g_2\rvert$	$17.28.39.$ $45.6X.ET$	$18.29.37.$ $45.6X.ET$	$2^7.3^{1+2}.2$	a3A$:ijk$; b3A$:lmn$; c3A$:opq$; d3A$:rst$; e6A$:msroqn$(6D8^2, 4A10^3); f6A$:jportk$; g6A$:iktsml$; h6A$:ilnqpj$; i6A$:jakglh$(6B2|4, 4A5|10^2); j6A$:kaihpf$; k6A$:iajftg$; l6A$:mbnhig$; m6A$:nblgse$; n6A$:lbmeqh$; o6A$:pcqerf$; p6A$:qcofjh$; q6A$:ocphne$; r6A$:sdtfoe$; s6A$:tdremg$; t6A$:rdsgkf$
6E	1	$(S_3^2\times A_4).2[.2^2]$	-1	25.34	$23.b_3$	$57.z_1^{39}$	$U_4(2)$	a4B$:bfhe$(6A21|26); b4B$:aegf$; c4B$:dgeh$; d4B$:chfg$; e5A$:ahcgb$(12B?); f5A$:bgdha$; g5A$:dfbec$; h5A$:ceafd$
6E	2	$3^{1+4}.2^{4+4}[.S_3\times A_4]$	-7	$c_2^{z_3}f_1$	$c_3^{z_1}f_2$	$c_1^{z_2}f_3$	$2^2\times 3^{1+2}.2$	hc$(6,2\omega-2)$: $a^{12}:a^6$(6D5^2, 2B7^3)
6E	3	$2^{4+4}.3[.2\times A_4]$	1	$1X.29.38.$ $46.5E.7T$	$18.2X.39.$ $46.5T.7E$	$14.36\|28.$ $57\|9X.ET$	$2^2\times 2^6.3^{1+2}.2$	hc$(6,2\omega-2)$: $a^4:b^6$(6B2^2, 2B7^3); $b^8:(ab)^3$(6D5|8, 4A10^3)
6E	4	$2^{1+4}.3^2.2[.6^2.6]$	-3	z_1z_2	$1X.29.38.$ $4E.56.7T$	$18.2X.39.$ $4T.56.7E$	$2^{1+8}.3^3.2^2$	hc$(6,6\omega)$: $a^{36}:a^6$(6C16^2, 6A27^3)
6E	5	$\frac{1}{2}(S_4\times S_3)[.6^2.6]$	-2	$14.b_3$	$19.28.37.$ $46.5X.ET$	$17.29.38.$ $45.6X.ET$	$O_8^+(2)$	hc$(6,6\omega)$: $a^{36}:a^6$(12C$?^2$, 4B22^3)
6E	6	$\frac{1}{2}(S_4\times S_3)$ $[.6^2.6,*]$	0	$14.b_3$	$19.28.35.$ $46.7X.ET$	$17.29.36.$ $45.8X.ET$	$O_8^+(2)$	hc$(6,6\omega)$: $a^{36}:a^6$(15A$?^2$, 6A27^3)
6E	7	$3^2.2[.6^2.6]$	1	$(17.26.35.48.$ $9X.ET)^{z_2.48}$	$(15.27.36.48.$ $9X.ET)^{z_2}$	$01\bar{1}001\bar{1}0.$ $\bar{1}\bar{1}111011$	$O_8^+(3)$	hc$(6,6\omega)$: $a^{36}:a^6$(9A$?^2$, 4B22^3)
6F	1	$2^{1+8}.L_2(8).3$	8	AB	$AC.BD$	$CE.DF$	$2\times S_4$	a2B$:bc$; b3C$:cac$; c4B$:babcT'$(4B3|5)

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
6F	2	$2\times 2^4.A_8$	0	d_1f_1	$\lvert b_1e_1\rvert$	$\lvert b_2e_2\rvert$	$2\times S_4$	a2B : bc; b3C : cac; c4A : $babcT'$(4C1 $\mid$ 2)
6F	3	$2^{1+8}.(3\times S_3)[.2]$	0	d_1f_2	$\lvert b_1e_1\rvert$	$\lvert b_2e_2\rvert$	$2^5.S_3$	a3C : cde; b3C : edc; c4A : $aebd$(4C1 $\mid$ 4); d4B : $acbe$(4B3 $\mid$ 6); e4B : $adbc$
6F	4	$2^{1+4}.(A_5\times A_4)[.2]$	0	$(\alpha,\gamma)(\beta,\delta)$	$(\gamma,-\epsilon)(\delta,\zeta)$	$(\beta^{\omega},\gamma)(\delta^{\epsilon}\zeta)$	$2\times A_5$	a2B : de; b3C : cde; c3C : edb; d5A : $ceaeb$(10B?); e5A : $bdadc$
6F	5	$L_2(8).3$	2	AB	$BC.DE$	$CD.EF$	$3S_6$	a2B : ef; b3C : gch; c4B : $bgdh$(3C1 $\mid$ 3); d4B : $cgfh$(6A16 $\mid$ 21); e5A : $hfafg$(6F5); f6A : $eaehdg$(4B12 $\mid$ 16, 4B5 $\mid$ 19^2); g6A : $bhefdc$(15A? $\mid$?, 6A10 $\mid$ 16 $\mid$ 21); h6A : $cdfegb$
6F	6	$2^3.L_3(2)[.2^2]$	4	$(\alpha,\beta+x)(\epsilon,\zeta)$ where x is $2(i-j)$	$\alpha^{ij}.\beta^{ik}.\gamma^{jk}.$ $(\delta,\epsilon^{jk}).\zeta^{jk}$	$\alpha^{jk}.(\beta,\gamma^{jk}).$ $\delta^{jk}.\epsilon^{ij}.\zeta^{ik}$	$2^6.3^{1+2}.2^2$	a2B : ef; b2B : gh; c4A : $iokm$(4C7 $\mid$ 7); d4A : $jpln$; e6A : $fafriq$(12C? $\mid$?, 6A3 $\mid$ 27^2); f6A : $eaeqjr$; g6A : $hbhtks$; h6A : $gbgslt$; i6A : $ocmqer$(6A10 $\mid$ 32, 6A18 $\mid$ 27^2); j6A : $pdnrfq$; k6A : $mcosgt$; l6A : $ndpths$; m6A : $icktpq$(12C? $\mid$?, 3A5 $\mid$ 4^2); n6A : $jdlsor$; o6A : $kcirns$; p6A : $ldjqmt$; q6A : $eimpjf$(6A32^2, 3C3 $\mid$ 3^2); r6A : $fjnoie$; s6A : $gkonlh$; t6A : $hlpmkg$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	1	$S_4(4).2$	−5	12.36	23.47	34.51	$L_3(2)$	$a3\text{A}:baTb$; $b4\text{B}:abSba(3\text{A}2^2)$
7A	2	$\frac{1}{2}S_5 \wr 2$	−17	14.23	25.34	16.27	A_7	$a3\text{A}:cde$; $b3\text{A}:edc$; $c4\text{B}:aebd(4\text{B}2\|19)$; $d5\text{A}:eacbe(7\text{A}3)$; $e6\text{A}:bcadeTd(4\text{B}13\|17, 5\text{A}3\|1^2)$
7A	3	$\frac{1}{2}S_5 \wr 2$	1	14.23	25.34	16.37	A_7	$a3\text{A}:bde$; $b3\text{A}:eda$; $c4\text{B}:decTe(6\text{A}21\|21)$ $d5\text{A}:becea(7\text{A}2)$; $e6\text{A}:adccdb(5\text{A}4^2, 3\text{A}3\|2^2)$
7A	4	$2^{4+4}.3^2.2$	−5	12.67	$(16.38)^{z_2}$	15.26.37. 48.9$X.ET$	$2^6.L_3(2)$	$a3\text{A}:ced$; $b4\text{B}:deec(6\text{A}19^2)$; $c4\text{A}:adbe(6\text{C}2\|7)$; $d4\text{A}:ebca$; $e6\text{A}:dacbeTb(7\text{A}1\|4, 4\text{A}5\|9^2)$;
7A	5	$2^6.S_5[*]$	−13	12.35	12.34.56. 78.9$X.ET$	18.23.45. 67.9$X.ET$	$2 \times 2^3.L_3(2)$	$a3\text{A}:ced$; $b4\text{A}:deec(6\text{C}4\|6)$; $c4\text{B}:adbe(6\text{A}4\|19)$; $d4\text{A}:ebca(6\text{A}5\|17)$; $e6\text{A}:dacbeTb(14\text{A}?\|?, 4\text{A}5\|8\|9)$
7A	6	$2 \times S_6[*]$	−11	$EW.GK.IV.$ $JR.LP.MU.$ $QT.XY$	$EI.HJ.KQ.$ $LS.NW.PU.$ $RY.TZ$	$EI.GH.KS.$ $LW.NQ.OT.$ $PX.VY$	$2.M_{21}$	$a3\text{A}:bdc$; $b4\text{B}:aced(5\text{A}1\|6)$; $c4\text{A}:deba(10\text{A}?\|?)$; $d5\text{A}:ecabe(10\text{A}?)$; $e5\text{A}:cdeTdb(14\text{A}?)$
7A	7	$5^2.4.2^2[.3]$	−15	$\begin{matrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{matrix}$	$\begin{matrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{matrix}$	$\begin{matrix} 2 & 0 & 2\overline{\omega\theta} \\ 0 & -1 & 0 \\ 2\omega\theta & 0 & -2 \end{matrix}$	$U_3(5)$	$a3\text{A}:hdi$; $b3\text{A}:ieg$; $c3\text{A}:gfh$; $d5\text{A}:eiahf(8\text{C}?)$; $e5\text{A}:fgbid$; $f5\text{A}:dhcge$; $g6\text{A}:fchibe(21\text{A}?^2, 4\text{B}2\|19^2)$; $h6\text{A}:daigcf$; $i6\text{A}:ebghad$

$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	8	$2\times A_6[.2]$	-17	$CD.GH.KP.$ $LO.MQ.NR.$ $UV.YZ$	$CD.IJ.KL.$ $QR.SZ.TY.$ $UW.VX$	$FO.GW.IU.$ $JM.LV.NZ.$ $PR.TX$	$2.M_{21}.2$	a3A : hfi; b3A : igh; c4A : $dfeg$(6A7|22); d4A : $cgif$(10A?|?); e4A : $cfhg$; f6A : $ecdiah$(14A?|?, 6A4|18^2); g6A : $dcehbi$; h6A : $aibgef$(8C?|?, 4A16|5^2); i6A : $bhafdg$
7A	9	$\frac{1}{2}S_4^2$	-11	$(15.23)^{z_2}$	$(25.34)^{z_2}$	$15.26.37.$ $48.9X.ET$	$2^6.A_8$	a3A : hbi; b4A : $ahgi$(12C?|?); c4A : $dfeg$(8B?|?); d4A : $cghf$(6C7|15); e4A : $cfig$; f5A : $iecdh$(15A?); g6A : $dceibh$(12A?|?, 6A11|19^2); h6A : $baifdg$(7A3|4, 6A4|22|32); i6A : $abgefh$
7A	10	$2\times 3^2.D_8$	-11	00001000. 00000100	$\bar{1}\bar{1}001100.$ $\bar{1}1001\bar{1}00$	$\bar{1}11\bar{1}0000$ $\bar{1}10\bar{1}0100$	$2.U_4(3)$	a3A : hci; b3A : feg; c4A : $ahdi$(10A?|?); d5A : $gichf$(12B?); e5A : $igbfh$(8B?); f5A : $gdheb$(14A?); g5A : $beidf$; h6A : $aiefdc$(9A?|?, 5A1|6|11); i6A : $acdgeh$
7A	11	$2\times S_5[.2,*]$	-13	$KQ.LR.MO.$ $NP.SY.TZ.$ $UW.VX$	$GS.HU.IV.$ $JT.OW.PZ.$ $QX.RY$	$EL.FP.IO.$ $JK.MY.NU.$ $QV.RZ$	$2M_{22}$	a3A : dfh; b3A : egi; c4A : $dhei$(6C12|14); d5A : $ahcif$(11A?); e5A : $bichg$; f5A : $adigh$(8B?); g5A : $behfi$; h6A : $afgecd$(10A?|?, 5A1|6|10); i6A : $bgfdce$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	12	$2^4.D_{12}[.2,*]$	−9	$DE.HI.LM.$ $PQ.SW.TY.$ $UX.\overline{VZ}$	$EF.IJ.\overline{MN}.$ $QR.SW.TX.$ $UZ.VY$	$FJ.GH.LO.$ $\overline{MV}.\overline{NZ}.QU.$ $RY.SX$	$2^{10}.A_7$	a3A : ceh; b3A : dfi; c4A : $ahge$(8B?|?); d4A : $bigf$; e5A : $acgih$(14A?); f5A : $bdghi$; g6A : $chfdie$(14A?^{2}, 4B11|15|22); h6A : $aeifgc$(8A?|?, 5A1|6|11); i6A : $bfhegd$
7A	13	$2^4.D_{12}[.2,*]$	−15	$DE.HI.LM.$ $PQ.\overline{SW}.TY.$ $UX.\overline{VZ}$	$EF.IJ.\overline{MN}.$ $QR.SW.TX.$ $UZ.VY$	$FJ.GH.LO.$ $NY.OV.PR.$ $QT.SU$	$2^{10}.A_7$	a3A : egh; b3A : fgi; c4A : $dieh$(12C?|?); d4A : $chfi$; e5A : $ahcig$(14A?); f5A : $bidhg$; g6A : $aeibfh$(7A3^2, 4B2|15|22); h6A : $agfdce$(10A?|?, 6A4|19|26); i6A : $bgecdf$
7A	14	$2\times S_4[*]$	−15	$ET.FX.\overline{IW}.$ $JS.MV.NY.$ $QU.\overline{RZ}$	$EI.GJ.KW.$ $LQ.NS.OZ.$ $RV.UX$	$FX.GO.HS.$ $\overline{JL}.\overline{KQ}.NT.$ $PU.\overline{WZ}$	$2^{11}.M_{22}$	a3A : ehi; b4A : $egfh$(8B?|?); c4A : $dfgi$(10A?|?); d4A : $cihf$(12C?|?); e5A : $aigbh$(11A?); f5A : $bgcdh$(22A?); g5A : $beicf$(22A?); h6A : $aebfdi$(14A?|?, 5A1|6|7); i6A : $ahdcge$(10A?|?, 6A4|22|28)
7A	15	$2\times D_8$	−13	c_1c_2	$b_1c_3^{b_3d_3z_1c_3}$	$c_1^{z_1z_3x}x^{z_3c_2c_1}$ where x is $c_3^{z_2c_1b_1}$	$2^{2+20}.U_4(3)$	a3A : eif; b4A : $cgdh$(12C?|?); c4A : $bhig$(8B?|?); d5A : $fhbge$(18C?); e5A : $afdgi$(14A?); f5A : $aihde$; g5A : $bcied$(16A?); h5A : $cbdfi$; i6A : $faegch$(9A?|?, 6A4|26^2)

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	16	$3^2.2[.2]$	-14	$(15.26.37.$ $48.9X.ET)^{z_2}$	$000\bar{1}1101$ $0\bar{1}110001$	11001100 $1\bar{1}00\bar{1}100$	$O_8^+(3)$	$a4\mathrm{A}:bhci(10\mathrm{A}?\mid?)$; $b4\mathrm{A}:aifh(8\mathrm{B}?\mid?)$; $c4\mathrm{A}:ahgi$; $d4\mathrm{A}:eghf(12\mathrm{C}?\mid?)$; $e4\mathrm{A}:dfig$; $f5\mathrm{A}:dhbie(18\mathrm{B}?)$; $g5\mathrm{A}:eichd$; $h6\mathrm{A}:cabfdg(15\mathrm{A}?\mid?,6\mathrm{A}11\mid 22^2)$; $i6\mathrm{A}:bacgef$
7A	17	$2^2.M_{21}[.S_3]$	-17	$12.34.56.$	$12.38.45.$	$13.25.67.$	$2^2\times L_3(2)$	$a4\mathrm{A}:bihc(6\mathrm{A}18^2)$; $b4\mathrm{A}:cgia$; $c4\mathrm{A}:ahgb$; $d4\mathrm{A}:fhie$; $e4\mathrm{A}:digf$; $f4\mathrm{A}:eghd$; $g6\mathrm{A}:chfeib(14\mathrm{A}?^2,2\mathrm{B}7\mid 4^2)$; $h6\mathrm{A}:aidfgc$; $i6\mathrm{A}:bgedha$
7A	18	$2^6.3^2.2[.S_3,*]$	-17	$13.4X$	$(17.26.3E.$ $4X.59.8T)^{b_3}$	$(15.2T.3E.$ $49.68.7X)^{z_2}$	$2^{3+6}.L_3(2)$	$a4\mathrm{A}:bihc(6\mathrm{C}7^2)$; $b4\mathrm{A}:cgia$; $c4\mathrm{A}:ahgb$; $d4\mathrm{A}:fhie$ $e4\mathrm{A}:digf$; $f4\mathrm{A}:eghd$; $g6\mathrm{A}:chfeib(7\mathrm{A}4^2,2\mathrm{B}22\mid 11^2)$; $h6\mathrm{A}:aidfgc$; $i6\mathrm{A}:bgedha$
7A	19	$\frac{1}{2}(S_4\times S_3)[.S_3]$	-13	$(4X.57)^{b_3}$	$16.2X.37.$ $48.59.ET$	$17.26.38.$ $4X.59.ET$	$O_8^+(2)$	$a4\mathrm{A}:bihc(12\mathrm{C}?^2)$; $b4\mathrm{A}:cgia$; $c4\mathrm{A}:ahgb$; $d4\mathrm{A}:fhie$ $e4\mathrm{A}:digf$; $f4\mathrm{A}:eghd$; $g6\mathrm{A}:chfeib(8\mathrm{B}?^2,6\mathrm{A}32\mid 11^2)$; $h6\mathrm{A}:aidfgc$; $i6\mathrm{A}:bgedha$
7A	20	$\frac{1}{2}(S_4\times S_3)[.S_3]$	-12	$25.b_3$	$16.23.48.$ $59.7X.ET$	$17.26.34.$ $59.8X.ET$	$O_8^+(2)$	$a4\mathrm{A}:bihc(8\mathrm{B}?^2)$; $b4\mathrm{A}:cgia$; $c4\mathrm{A}:ahgb$; $d4\mathrm{A}:fhie$ $e4\mathrm{A}:digf$; $f4\mathrm{A}:eghd$; $g6\mathrm{A}:chfeib(10\mathrm{A}?^2,4\mathrm{B}22\mid 11^2)$; $h6\mathrm{A}:aidfgc$; $i6\mathrm{A}:bgedha$

$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	21	$2^2 \times D_8[.2]$	-15	$c_2 b_3^{c_3 z_2 c_1 b_1}$	$c_3^{z_1} c_1^{z_2 c_3 z_1 c_2}$	$\pi z_3 z_3^{c_1 z_2 c_2 z_1 z_3^*}$	$2^{3+18}.M_{21}$	a4A : $begi$(10A?$\mid$?); b4A : $aihe$; c4A : $dfhi$; d4A : $cigf$; e5A : $bhfga$(14B?); f5A : $dgehc$; g5A : $fdiae$(20A?); h5A : $ebicf$; i6A : $agdchb$(14A?^{2}, 4B15$\mid 11^2$)
7A	22	$D_{12}[.2]$	-14	$49.3X$	$(38.7X)^{z_3}$	$ff^{(67)(1X)}$	$2.Fi_{22}$	a4A : $begi$(12C?$\mid$?); b4A : $aihe$; c4A : $dfhi$; d4A : $cigf$; e5A : $bhfga$(21A?); f5A : $dgehc$; g5A : $fdiae$(16A?); h5A : $ebicf$; i6A : $agdchb$(14A?^{2}, 6A26$\mid 11^2$)
7A	23	$D_{12}[.2]$	-13	$89.3X$	$(25.89)^{z_3}$	$(45.f)^{(48)}$	$2.Fi_{22}$	a4A : $begi$(8B?$\mid$?); b4A : $aihe$; c4A : $dfhi$; d4A : $cigf$; e5A : $bhfga$(16A?); f5A : $dgehc$; g5A : $fdiae$(21A?); h5A : $ebicf$; i6A : $agdchb$(11A?^{2}, 6A26$\mid 11^2$)
7A	24	$2^2 \times L_3(2)[.S_3]$	-19	$GX.HZ.IY.$ $JW.KU.LT.$ $MV.NS$	$FJ.GI.KS.$ $LR.MW.OU.$ $QY.TZ$	$FU.GZ.HQ.$ $JN.KY.MP.$ $RW.TV$	$2^2.M_{21}$	a4A : $eihf$(6A23^2); b4A : $fhgd$; c4A : $dgie$; d5A : $fbgce$(14A?); e5A : $dciaf$; f5A : $eahbd$; g5A : $icdbh$; h5A : $gbfai$; i5A : $haecg$
7A	25	$2^2 \times S_4[.S_3,*]$	-11	$\overline{GX}.\overline{HZ}.\overline{IY}.$ $JW.KU.LT.$ $MV.NS$	$FJ.\overline{GI}.KS.$ $LR.MW.OU.$ $\overline{QY}.TZ$	$FU.\overline{GZ}.\overline{HQ}.$ $JN.KY.MP.$ $RW.TV$	$2^{11}.M_{21}$	a4A : $eihf$(6C7^2); b4A : $fhgd$; c4A : $dgie$; d5A : $fbgce$(14A?); e5A : $dciaf$; f5A : $eahbd$; g5A : $icdbh$; h5A : $gbfai$; i5A : $haecg$
7A	26	$D_{10}[.S_3]$	-16	$(4X.6T)^{du.x}$ where x is $71T.2E.5X6$	$(19.26.3T.$ $4X.5E.78)^{du}$	$(1X.26.3E.$ $4T.59.78)^{du}$	HN	a4A : $eihf$(10A?^{2}); b4A : $fhgd$; c4A : $dgie$; d5A : $fbgce$(25A?); e5A : $dciaf$; f5A : $eahbd$; g5A : $icdbh$; h5A : $gbfai$; i5A : $haecg$

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$\boldsymbol{abc}$	#	$C_M\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	wt	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\langle \boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\rangle$	net
7A	27	$2^2[.S_3]$	-15	$(c_3^{b_3}d_1)^x$ where x is $b_1^{ac_1}$	$\|f_2a_2\|b_3fz_3.$ $(b_1d_1d_2d_3fz_1)^*$	$\|f_3a_3\|b_1b_3.$ $d_3z_1z_3.$ $(b_1d_1d_2d_3fz_1)^*$	$2^2.{}^2E_6(2)$	a4A : $eihf$(12C?^{2}); b4A : $fhgd$; c4A : $dgie$; d5A : $fbgce$(38A?); e5A : $dciaf$; f5A : $eahbd$; g5A : $icdbh$; h5A : $gbfai$; i5A : $haecg$
7A	28	$2^2[.S_3]$	-14	$(z_1b_3)^{c_3ab_1c_1}$	$\|f_2a_2\|b_3fz_3.$ $(b_1d_1d_2d_3fz_1)^*$	$\|f_3a_3\|b_1b_3.$ $d_3z_1z_3.$ $(b_1d_1d_2d_3fz_1)^*$	$2^2.{}^2E_6(2)$	a4A : $eihf$(8B?^{2}); b4A : $fhgd$; c4A : $dgie$; d5A : $fbgce$(13A?); e5A : $dciaf$; f5A : $eahbd$; g5A : $icdbh$; h5A : $gbfai$; i5A : $haecg$
7B	1	$7^2.6A_4$	4	$AB.CF$	$AE.CD$	$BC.DG$	$L_3(2)$	a3C : $baTb$; b4B : $abSba$(3C1^2)
7B	2	$7.3[.7^2.6]$	0	12|01232 2313100. 12|00102 3232311	45|10026 4799583. 45|10026 4899375	41|01245 0354213. 41|01241 3354250	He	hc$(7,7\omega)$: a^{49} : a^6(12C?^{2}, 5A8^3)
7B	3	$7.3[.7^2.6]$	-1	12|01232 2313100. 12|00102 3232311	45|10026 4799583. 45|10026 4899375	$1X$|01234 $7X$68E95. 41|01245 3103524	He	hc$(7,7\omega)$: a^{49} : a^6(15A?^{2}, 3C3^3)

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