LETTERS TO THE EDITOR

SOME COMMENTS ON A PAPER BY H. J. WEINER

(Sequential random packing in the plane. *J. Appl. Prob.* **15** (1978), 803–814)

Dear Editor,

Has the Palásti conjecture been proved?: a criticism of a paper by H. J. Weiner

Weiner (1978) claims that he has verified the Palásti conjecture (Palásti (1960)) in the plane and, more generally, in the n-dimensional space. In opposition to this, I have great doubt about his conclusions. The purpose of this note is to point out the most fundamental errors in the paper of Weiner (1978). In this note, I mention only Model I (Rényi's car parking problem) because Weiner's conclusions on Model II and on random size cars are based essentially on the same errors as those contained in the conclusions of Model I.

Firstly, I would like to point out an error in the statement of Lemma 2, which Weiner calls a key lemma. He states that the $\alpha \times \beta$ cars in the $a \times b$ rectangle with coordinates $(0,0)$, $(0,b)$, $(a,0)$, (a,b) $(\alpha, \beta \leqslant a, b)$ intersect line segment l, which combines $(0, b-\beta)$ to $(a, b-\beta)$, in segments *in accord with* a one-dimensional law of Model I. In the proof of this lemma, he states that the horizontal placement and parking of cars on l is *independent* of all other parked cars in the rectangle and depends *only* on the x-coordinate. I show that this is incorrect.

Let us consider, in the rectangle, a strip whose width is β and whose midparallel coincides with the segment l. The $\alpha \times \beta$ cars whose centres are inside this strip intersect the segment l without fail. Let us assume that, in the course of the parking process, a certain number of cars is already placed in the rectangle. The room which is left for the centres of the cars to be placed afterwards, which I call a residual space, is the space in the rectangle deleted by $(2\alpha) \times (2\beta)$ rectangles, whose centres coincide respectively with those of already placed $\alpha \times \beta$ cars. Therefore, at a certain stage of the parking process, the residual space inside the above-mentioned strip may be deleted by several

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$(2\alpha)\times(2\beta)$ rectangles. The residual space inside the strip partly has a width smaller than β. Accordingly, if the x, y-coordinates are chosen i.i.d. uniformly, the placement of car centres in the residual space inside the strip depends on the width, i.e., the probability of car placement is smaller in the region where the width is narrow than in the region where it is broad and vice versa. Therefore, the placement of cars certainly *depends* on the other parked cars and depends on *both* the x and y coordinates, contrary to Weiner's statement. Moreover, it is easily seen from the above argument that the expectation of the number of parked cars which intersect the segment l is less than $M_\alpha(a)$, the mean number of cars of length α parked in an interval of length a in accord with Model I.

Secondly, I must point out a serious fault in Lemma 3. Weiner states in (2.7a) that, for $a > 2\alpha$, $a^{-1}M_\alpha(a)$ is a monotone *decreasing* function of a. However, this cannot be concluded by taking derivatives of $a^{-1}M_\alpha(a)$ and $M_\alpha(a)$ and by checking their signs. Furthermore, (2.7a) is incorrect. It was shown by Rényi (1958) that, asymptotically $(a\to\infty)$, the relation $\alpha a^{-1}M_\alpha(a) = \eta-(1-\eta)\alpha a^{-1}+o(a^{-n})$ $(n \geqq 1)$ holds; that means that $a^{-1}M_\alpha(a)$ is asymptotically a monotone *increasing* function of a because of the relation $\eta < 1$. Accordingly, the relations (2.5b) or (2.8a), i.e., the monotone decrease of $M(a,b)/ab$, the density in the plane, cannot be concluded even though Lemma 2 were correct. I note here that the inverse relation (monotone increase of $M(a,b)/ab$) also cannot be theoretically derived from Weiner's 'row formation' even if (2.7a) is correctly improved. It is because Lemma 2 is invalid and because, even if Lemma 2 is corrected as I mentioned in the last part of the previous paragraph, the inequality between $a^{-1}M_\alpha(a)$ and the parking density on the segment l does not result in the concerning monotonicity property of $M(a,b)/ab$. The inequalities (2.6a) and (2.6b), accordingly, have no theoretical basis. For that reason, the derivation of the inequalities in Lemma 4 is wrong. As a result of these discussions, the statement of Theorem 1, i.e., $\lim_{a,b\to\infty}\alpha\beta(ab)^{-1}M(a,b)=\eta^2$, loses its theoretical basis. Consequently, I should say that the validity of the Palásti conjecture has not yet been proved.

References

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